



# GREEN RESEARCH GRANTS

BY  
SEAN D'EVELYN

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UNIVERSITY OF HAWAII AT MANOA  
2424 MAILE WAY, ROOM 540 • HONOLULU, HAWAII 96822  
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## **Abstract**

Environmentally friendly technologies typically yield social returns far in excess of the private returns to the innovating firm. Governments issue grants to fund such projects in an attempt to increase their supply, but completion is uncertain and many projects never yield a successful innovation. This paper examines the case where each project has a probability of being feasible which is known to the firm, but unknown to a benevolent grant authority. The grant authority therefore creates a menu of grants in such a way as to elicit truthful revelation of firms' types. More feasible projects indeed receive more funds than less feasible projects, but are expected to work much harder and finish faster.

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**Sean D'Evelyn**

University of Hawai'i - Mānoa

E-mail: [sdevelyn@hawaii.edu](mailto:sdevelyn@hawaii.edu)

# 1 Introduction

Many of the environmental problems facing the world today, most notably global warming, are generally thought to be problems that are unable to be solved fully with the current state of technology.<sup>1</sup> Environmentally sound (ie “green”) technological advancement is therefore imperative. Under standard economic theories, however, the market underproduces green research and development due to the relatively large gap between the potential social surplus from such innovations and the private profits the innovating firms reap. Indeed, while the innovating firm bears all the costs for its research effort, the benefits from its invention are shared with the rest of society due to the public nature of both environmental goods and the knowledge from the innovative process. This paper examines how a grant authority may induce more efficient innovative efforts when innovation is an inherently uncertain process that does not always lead to success.

Each year the US federal government awards roughly \$500 billion dollars in grant money.<sup>2</sup> While much of this money does go towards paying for infrastructure and services, a substantial amount is given to academic as well as private institutions for continuing research. These are federal dollars that pay for the innovative efforts in academia and in private firms. The money is not returned if the project proves unsuccessful, and an inventor still owns the patent rights to any resulting innovation.

In this way, research grants are unique among the policy tools used by the government to help bridge the gap between the social optimal level of innovation and the free market level. While inventions made at national laboratories are the property of the funding government, inventions made through research funded by a grant still belong to the inventing agent. When the government strengthens patent law, provides stricter environmental regulations, or even funds an innovation prize, it attempts to procure more innovation by increasing the returns to successful innovations. With grants, the benefits upon innovation remain the same, the firms simply have more funds available to do the research.

In this paper, not all projects yield successful innovations. Projects differ in terms of their feasibility - the probability that a project could yield a meaningful innovation if given enough time and effort. Unlike models where innovation occurs when a knowledge stock crosses a threshold, uncertain feasibility makes projects less worthwhile over time rather than more, and projects may be terminated if they take too long. The probability that a project is feasible is known to the firm and is reported to the grant authority who

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<sup>1</sup>Cases of past environmental problems that required large technological development include the control of chlorofluorocarbons (Barrett 1992) and sulfur dioxide (Popp 2006).

<sup>2</sup>According to [www.grants.gov](http://www.grants.gov).

then sets and agrees to pay for a certain path of effort.

In a socially optimum setting, both firms and the government learn to update their beliefs of a firm's likelihood of success in continuous time. The firm's optimum level of effort, then, is a function only of the current belief of its feasibility. In the case of incomplete information, the social optimal effort path may be implemented under some parameter values. In general, though, the government will have to make tradeoffs to ensure that firms are willing to truthfully reveal their type.

In the sections that follow, section 2 reviews the most relevant literature to the current problem. Section 3 goes over the model with and without complete information, while section 5 provides the conclusions and discussion.

## 2 Related Literature

The need for government intervention in research and development in general and environmental research in particular, is well documented (see, for example, Jaffe et al. (2005) for an overview). Much attention has been made at looking at the proper design of patent laws to help incentivize innovation at the least cost to society. In the environmental literature, many articles have looked at the use of strict environmental standards to encourage technological progress.

However, it seems as though in the United States and most OECD countries, the patent system is not going to change significantly any time soon, and any revisions are slow moving. Furthermore, any changes in environmental law may alter the incentives for firms to invest in green technologies, but it will not eliminate the usefulness of grants. The practical manager of the grant authority takes existing patent law and environmental regulations as given.

The present paper builds upon two distinct strands of literature. The first is the environmental literature that examines the production of public goods. One seminal paper from this strand that is closest to the present paper is Tsur and Zemel (2002) where a government agency attempts to produce an innovation or another public good by eliciting contracts (grant applications) from a pool of applicants.<sup>3</sup> The government agency is neither able to observe firms types nor their effort level, but rather elicits truth telling behavior by imposing deadlines until completion. If the firm takes longer than the deadline to innovate, then a severe penalty is incurred.

The current paper follows Tsur and Zemel in that it models innovation as a stochastic

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<sup>3</sup>Tsur and Zemel's model of deadline-setting is further characterized in Toxvaerd (2007).

process that takes place in continuous time. Because the government cannot observe the productivity of firms directly, it is forced to structure contracts in such a way as firms willingly divulge the information themselves. However, in their mechanism design, the authors allow the government to levy strict fines on firms that do not meet their deadlines. While this may be relatively common for construction projects, granting agencies rarely have the same ability. Typically, the worst punishment a grant authority has at its disposal for a non-performing agent is simply the cessation of funds. The current paper assumes the grant authority is unable to fine non-performing firms.

Furthermore, Tsur and Zemel models the innovative effort as slowly building knowledge capital. Knowledge is accumulated until it reaches a critical level at which point the project is complete. In their model, while the timing of success may be uncertain, the project is guaranteed to succeed eventually. Furthermore, there is no incentive to stop a project that has already accumulated some capital. The current paper does not model knowledge capital and past effort does not increase the likelihood of innovating.

The current paper also builds on the finance literature that examines the funding of entrepreneurial endeavors. One particularly relevant example from this literature is Bergeman and Hege (2005). In their model, a venture capitalist invests in a research and development project. The venture capitalist is only able to supply funding for innovation that is renegotiation proof, that is, the funding is only a function of the current probability that the project is feasible. They find that, like Tsur and Zemel, the funding capitalist will cut off funding early in order to keep firms from seeking excess rents.

Like Bergeman and Hege, the current paper looks at an innovative process that not only is uncertain in its timing, but also in its very feasibility. Each project has a probability that no matter how much effort is exerted, the project will not yield a successful innovation because the project is infeasible. An example of such a project might be the projects funded during the 1970s to attempt to make oil from shale. Despite the massive amounts of grant money spent on these projects, no one has yet invented a cost effective way to make the transformation.<sup>4</sup>

Unlike Bergeman and Hege, the current paper's principal (the grant authority) does not have the power to alter the share of the rewards upon successful innovation that it and the agent receive. Grants typically end when the project is completed and no additional

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<sup>4</sup>In 2001, the National Research Council released a report on a subset of innovative projects funded by the Department of Energy from 1978 to 2000, including research meant to produce oil from shale. They found that a majority of their case studies did not yield innovations substantial enough to justify the research costs. However, 6 of the 17 case studies were so successful, that they could justify the entire portfolio of investments.

funds are given as a prize for innovating. Furthermore, Bergeman and Hege's results rely on the creation of renegotiation proof contracts. In the current paper, the government does not need to worry about renegotiation, as it is effectively the only source of funding that considers the social returns to innovation.

### 3 The Regulation Problem

#### 3.1 Complete Information

A firm is endowed with a project that has the potential to create large social surplus as well as some private surplus for the innovating firm. The present value of the stream of benefits generated from successful innovation is denoted  $R$ . We assume that either  $R$  is known with certainty or that both the firm and the government have the same estimation for  $R$  that does not vary with time. The present value of the profits generated by the firm is denoted  $sR$  with  $s \in (0, 1)$  denoting the share of the surplus that the firm enjoys.

In order to successfully innovate, the firm must engage in innovative effort. The level of effort for a firm at time  $t$  is denoted  $y_t$  and is measured in dollars. In addition, the firm also pays for the disutility of effort in the form of less comfortable and motivated employees. The value of this disutility, in dollars, is denoted  $\psi(y_t) \equiv \psi_t$ , with  $\psi(\cdot)$  strictly convex. Furthermore, we assume  $\psi(0) = \psi'(0) = 0$ . The total cost per unit of innovative effort becomes:  $y_t + \psi_t$ .

If the project was perfectly feasible, each dollar of effort would yield an additional  $h$  probability of successful innovation given that innovation had not yet occurred. However, in this model not all projects are feasible, as some projects will never yield a working product no matter how much effort is spent. The probability that the project is feasible is denoted  $p$ , and is known to the firm. Together, this means that the unconditional probability of innovating at a given time,  $t$ , can be written:

$$pdf(y; p) = py_t e^{-\int_0^t hy_\tau d\tau} \quad (1)$$

and the unconditional probability that the project not yet successful by time  $t$  is:

$$CDF(y; p) = 1 - p + pe^{-\int_0^t hy_\tau d\tau} \quad (2)$$

In order to simplify our notation slightly, we will be using the term  $A_t = \int_0^t hy_\tau d\tau$ , which measures the cumulative innovative effort the firm has exerted prior to time  $t$ . Even though the cumulative effort appears in the objective function, it is quite different from a knowledge stock, as a higher  $A_t$  actually decreases the likelihood of innovating rather

than increasing it. Practically, it means the more effort is spent without success, the less likely the project was feasible to begin with.

Absent government intervention, the firm seeks to maximize expected profits, equal to:

$$\max_{y_t} \int_0^{\infty} ((-y_t - \psi_t)(1 - p + pe^{-A_t}) + phy_t e^{-A_t} sR) e^{-rt} dt \quad (3)$$

$$\begin{aligned} \text{subject to: } \quad \dot{A}_t &= hy_t \\ A_0 &= 0 \end{aligned}$$

or, from some other time in the future:

$$\max_{y_\tau} \int_t^{\infty} ((-y_\tau - \psi_\tau) \frac{(1 - p + pe^{-A_\tau})}{(1 - p + pe^{-A_t})} + sR \frac{phy_\tau e^{-A_\tau}}{(1 - p + pe^{-A_t})}) e^{-r(\tau-t)} d\tau \quad (4)$$

$$\begin{aligned} \text{subject to: } \quad \dot{A}_\tau &= hy_\tau \\ A_t &= 0 \end{aligned}$$

Equations (3) and (4) simply state that until the project is completed, which is true with probability  $1 - p + pe^{-\int_0^t hy_\tau d\tau}$ , the firm will continue to spend on effort and suffer the disutility of its effort. However, with probability  $phy_t e^{-\int_0^t hy_\tau d\tau}$  the project yields a successful innovation with a (present value equivalent) payoff of  $sR$  to the firm. Profit is then expected revenues minus expected costs.

In the case of no government intervention, the firm's profit maximizing strategy is to set  $y_t$  such that:

$$(-1 - \psi_t)(1 - p + pe^{-A_t})e^{-rt} + pe^{-A_t}(hsR)e^{-rt} + \lambda_t h = 0 \quad (5)$$

$$(-y_t - \psi_t + hy_t sR)pe^{-A_t - rt} = \dot{\lambda}_t \quad (6)$$

Where  $\lambda$  is the costate variable for  $A_t$ , our cumulative effort variable, and can be thought of as the shadow value of fruitless effort. The first equation states that each period effort is set such that the current value marginal cost of effort (the first term) is worth the marginal increase in probability of success in the current period (the second term), modified by the recognition that effort in this period decreases the likelihood of innovating in the future periods (the third term). The second equation gives us the equation of motion for the costate variable. The costate variable increases over time equal to the per period payoffs given the project is feasible (in present value terms) times the probability the project is feasible.

We shall refer to this effort path that solves the above two equations as the Business as Usual (BAU) path.

From a societal, First-Best Optimal (FBO) point of view, the objective function is:

$$\max_{y_\tau} \int_t^\infty ((-y_\tau - \psi_\tau) \frac{(1-p+pe^{-A_\tau})}{(1-p+pe^{-A_t})} + R \frac{phy_\tau e^{-A_\tau}}{(1-p+pe^{-A_t})}) e^{-r(\tau-t)} d\tau \quad (7)$$

$$\begin{aligned} \text{subject to: } \quad \dot{A}_\tau &= hy_\tau \\ A_t &= 0 \end{aligned}$$

The only difference between equation (3) and (7) is that while the firm values only its own private returns from the invention,  $sR$ , the government values the entire social benefit created,  $R$ .

In order to form a tractable answer to the above problem, we first take a look at optimal learning. In this model, the only observable outcome of research is whether or not an innovation is successful. In this way, the longer it takes to successfully innovate, the less likely it is that the project is feasible. We next define a firm's subjective feasibility at time  $t$  as the probability that it is feasible given initial feasibility and the amount of effort that was exerted prior to time  $t$ . The formula for this subjective feasibility,  $\hat{p}_t$  is given by:

$$\hat{p}_t \equiv \frac{pe^{-\int_0^t hy_\tau d\tau}}{1-p+pe^{-\int_0^t hy_\tau d\tau}} \quad (8)$$

$$\dot{\hat{p}}_t = -\hat{p}_t(1-\hat{p}_t)hy_t \quad (9)$$

Even if a project begins with a relatively high probability of being feasible, the more effort is exerted, the less certain the firm is that the project can be completed. With this new notation, we can thus reduce equation (4), above, to:

$$\max_{y_\tau} \int_t^\infty ((1-\hat{p}_t)(-y_\tau - \psi_\tau) + \hat{p}_t(-y_\tau - \psi_\tau + hy_\tau sR)) e^{-A_\tau} e^{-r(\tau-t)} d\tau \quad (10)$$

$$\begin{aligned} \text{subject to: } \quad \dot{A}_\tau &= hy_\tau \\ A_t &= 0 \end{aligned}$$

A similar equation can be written for the FBO, with the returns upon innovation being  $R$  instead of  $sR$ . This leads us to our first proposition.

**Proposition 1** *The Business as Usual and the First-Best Optimal effort at any point in time can be written as a function of the subjective feasibility,  $\hat{p}_t$ , at time  $t$ .*



*Proof:* Take any feasibility,  $p < 1$ , where the BAU (or FBO) effort is greater than zero. At time  $t > 0$ , the subjective probability declines to  $p' = \frac{pe^{-\int_0^t hy_\tau d\tau}}{1-p+pe^{-\int_0^t hy_\tau d\tau}}$ . From equation (10), above, we see that the the problem for the firm starting at time  $t$  is equivalent to the problem they would have faced if their initial feasibility was  $p'$ . ■

This proposition indicates that optimal policy from either the firm or the government's perspective can be determined solely on the basis of what the current likelihood that the project is feasible. Therefore, for the rest of our analysis of the BAU and FBO effort paths, we can simply compare the two at a given level of feasibility, and do not need to concern ourselves with the whole path.

**Lemma 1** *The Business as Usual and First-Best Optimum effort paths are continuous in time/feasibility and twice differentiable with respect to feasibility.*

*Proof:* The objective function and the growth of our state variable are both twice continuously differentiable. The objective function is strictly quasiconcave in effort, and once it is optimal to exert zero effort, it will always be optimal to exert zero effort. Provided the optimal path is unique, then by the dynamic envelope theorem (LaFrance and Barney 1991), effort paths will be twice continuously differentiable with respect to feasibility. Because effort can be written as a function of current subjective feasibility and subjective feasibility changes continuously with time, we know that effort will also be continuous in time. ■

**Lemma 2** *The costate variable,  $\lambda$ , is less than or equal to zero for all time/feasibilities for the Business as Usual (or First-Best Optimal) solution.*

*Proof:* From equation (6), we have an equation for  $\dot{\lambda}$ . We further know that as  $t \rightarrow \infty$ ,  $\lambda \rightarrow 0$ . Together, this gives us an equation for  $\lambda$ .

$$\lambda_t = - \int_t^\infty (-y_\tau - \psi_\tau + hy_\tau sR) pe^{-A_\tau - r(\tau-t)} d\tau \quad (11)$$

Since the integrand must always be nonzero (else the project would not be worth ANY effort),  $\lambda < 0$  and  $\dot{\lambda} > 0$ . ■

This makes sense, as subjective feasibility,  $\hat{p}$ , decreases as the state variable,  $A$ , rises, making  $A$  undesirable. Not only is  $\lambda$  negative, but it is equal to the entire value of the project as if it were feasible times the probability that it is indeed feasible. Note that this is larger in absolute value than the actual value of the project itself.

**Lemma 3** *The per period payoffs for the firm (government) is increasing in effort on the Business as Usual (First-Best Optimal) effort path.*

*Proof:* To prove that  $-y - \psi + hysR$  is increasing in  $y$  along the BAU path, we can see that its derivative,  $-1 - \psi' + hsR$ , is found in equation (5). The first term,  $(-y - \psi)(1 - p) < 0$  and, thanks to lemma 3, we know that  $\lambda h < 0$  as well. Therefore,  $-1 - \psi' + hsR$  must be greater than zero for the left-hand side to equal zero. ■

**Theorem 1** *The First-Best Optimum (or Business as Usual) effort levels are increasing in subjective feasibility.*

This theorem posits that the lower subjective feasibility is, the lower the current marginal benefits to effort and the higher its costs. The only way that it could be worthwhile to decrease effort with feasibility is if the shadow price of cumulative effort,  $\lambda$ , increased on the margin with feasibility by an amount greater than the entire surplus, which seems unlikely. For the remainder of the paper we will assume this theory to hold.

**Proposition 2** *The First-Best Optimum effort path is greater or equal than the Business as Usual effort path for any feasibility.*

*Proof:* Until a project is successful, both the FBO problem and the BAU problem face identical costs for any given level of effort. Upon successful completion, however, the FBO problem receives a payoff of  $R$ , while the BAU problem only receives a payoff of  $sR < R$ . Given this, the incentives for innovative effort are strictly larger in the *FBO* problem, leading to a higher level of effort. ■

It can be argued that for many environmental innovations, the distance between the private and public returns from invention are large. It is therefore in the interested of a socially-minded government agency to attempt to increase the innovative efforts for such projects beyond their BAU levels.

This is not to say that all socially beneficial innovations are worth funding, as detailed in Proposition 3

**Proposition 3** *There exist minimum feasibilities below which innovative effort for the project is reduced to zero under both Business as Usual and the First-Best Optimum.*

*Proof:* At the point funding is cut off,  $\lambda = 0$ , from equation (11). Given that, the optimum level of effort solves:

$$(1 - \hat{p})(-1 - \psi') + \hat{p}(-1 - \psi' + hR) = 0.$$

Since effort varies continuously with  $\hat{p}$ , this must mean that at the point at which feasibility would be cut off, exerting zero effort must be optimal. In other words,  $(1 - \hat{p})(-1) + \hat{p}(-1 + hR) = 0$ , or  $\hat{p} = \frac{1}{hR-1}$ . Similarly, the minimum feasibility to fund a project in the BAU scenario is  $\hat{p} = \frac{1}{hsR-1}$ . ■

Therefore, not only does the government have incentive to fund potentially feasible projects to increase effort beyond the BAU level, but it also has incentive to make sure that its funding is going to the most feasible projects.

### 3.2 Imperfect Information

In the case where firms accept research funds from the grant authority in exchange for producing more socially beneficial innovations, the problem becomes a bit different.

We assume the grant authority has access to one mechanism to encourage research. Under this mechanism, all grant money pays for innovative efforts, and the requested level of effort is tied to the grant and set by the government. Furthermore, once the firm agrees to accept a grant, the firm's effort levels are observable to the grant authority. This assumes firms are unable to redirect grant money to fund projects other than the project applied for.<sup>5</sup> We assume that firms are free to end the grant without penalty should required efforts become too high.

Assuming a firm never terminates a grant prematurely, the firm's profit function becomes:

$$\int_0^{\infty} ((-\psi_t)(1 - p + pe^{-At}) + phy_t e^{-At} sR) e^{-rt} dt \quad (12)$$

$$\begin{aligned} \text{subject to: } \quad & \dot{A}_{it} = hy_t \\ & A_0 = 0 \end{aligned}$$

Suddenly the monetary cost of the innovative effort is inconsequential to the firm as the government has agreed to pick up the tab. The firm does have some reason to limit its own innovative effort, as it still bears the disutility of its effort. We denote the preferred level of effort by the firm given its monetary costs are covered as the Fully-Funded Desired (FFD) effort path. Unsurprisingly, many of the propositions from section 3.1 that apply to the BAU and the FBO paths also apply to the FFD effort path.

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<sup>5</sup>Note that if the firm's efforts were still unobservable to the grant authority, the firm would exert exactly the same amount of effort as it would under business as usual and then redirect the rest of the funds elsewhere.

**Proposition 4** *The desired effort path when a firm is fully funded can be written as a function of its subjective feasibility, and is strictly increasing in subjective feasibility and therefore non-increasing in time. Furthermore, desired effort is positive as long as feasibility is non-zero.*

*Proof:* Equation (12) is almost identical to equation (3) in section 3.1, except it is missing a  $-y$  term. While this will have large implications for the level of desired funding, the problem from time  $t$  will also look the same for any two projects with the same subjective probability,  $\hat{p}$ , regardless of the initial feasibility and previous effort. Similarly, the costate variable for the new problem is still the discounted sum of expected benefits into the future, and thus will still lead to decreasing effort over time and thus increasing effort with respect to  $\hat{p}$ . We can also find that desired effort only goes to zero as  $\hat{p}$  goes to zero. ■

While these characteristics of the FFD path are useful, it is more important to look at the FFD effort path relative to both the FBO and the BAU effort paths.

**Proposition 5** *The desired level of effort for a firm when the monetary effort costs are covered is always larger than under Business as Usual. The Fully-Funded desired level of effort is also higher than the First-Best Optimum level of effort when feasibilities are low. However, for some parameter values, the desired level of effort may be lower than the First-Best Optimal for more feasible projects.*

*Proof:* Compared with the BAU objective function, the FFD case has monotonically smaller effort costs and identical returns to innovation. Therefore, optimal effort levels increase.

To see that FBO and FFD effort levels cross, remember that for any non-zero feasibility below  $\frac{1}{hR-1}$ ,  $y^{FBO} = 0$  and  $y^{FFD} > 0$ . Therefore, FFD effort levels are always below FBO effort levels for very low feasibility projects. To see that the reverse can be true when feasibilities are very high, consider the case where feasibility is perfect ( $p = 1$ ). In this case, feasibility never decreases and effort is constant. The per period benefits of effort under FBO is  $-1 - \psi' + hR$  and under FFD is  $-\psi' + hsR$ . When  $s < 1 - \frac{1}{hR}$ , which is arguably the case for most environmental innovations, then  $y^{FBO} > y^{FFD}$ . Because effort is continuous with respect to feasibility for both the FBO and the FFD problems, this implies that there must be a crossing point. ■

For very low levels of  $p$ , the firm does not pay much cost for its effort, but stands to gain substantially if their innovation is successful. Therefore, they desire more innovative

effort than the social optimal. As  $p$  increases, however, the government again desires more effort than the firm would prefer simply due to the relatively small share of the social surplus the firm sees as profits. The exact place where this switch occurs depends on the size of  $s$  relative to  $h$  and  $R$ .

[Insert Figure 1 about here]

Given asymmetric information, the government takes the firm's profit function as given and tries to create a menu of grants for each possible feasibility in such a way as to elicit truthful revelation of each project's type. Assuming the grant agency has access to non-distortional funds, its optimization problem is:

$$\max_{y_t} \int_0^\infty ((-y_t - \psi_t)(1 - p + pe^{-At}) + phy_t e^{-At} R) e^{-rt} dt \quad (13)$$

subject to:  $\dot{A}_t = hy_t,$   
 $\int_t^\infty ((-\psi_\tau) \frac{(1 - p + pe^{-A\tau})}{(1 - p + pe^{-At})} + sR \frac{phy_\tau e^{-A\tau}}{(1 - p + pe^{-A\tau})}) e^{-r(\tau-t)} d\tau \geq 0 \forall t,$

and  $p, \infty$  solves:

$$\max_{\tilde{p}, T} \int_0^T (-\psi(y_t(\tilde{p}))(1 - p + pe^{-\int_0^t hy_\tau d\tau}) + phy_t(\tilde{p}) e^{-\int_0^t hy_\tau d\tau} sR) e^{-rt} dt$$

The above objective function is the same as the FBO problem, but with the added individual rationality and incentive compatibility constraints. The first new constraint is the individual rationality constraint. It stipulates that the expected returns are greater than zero for all time. This is because firms are able to freely stop projects if too much is required of them. When this constraint is satisfied, even though a firm is able to cancel a government grant whenever they want, they never choose to. The second constraint requires that firms receive the most profit from truthfully revealing their own types.

We now ask whether implementing the first best policy is ever also incentive compatible when a project's feasibility is unobservable to the grant authority. The somewhat surprising answer is yes, although in rather limited circumstances.

**Proposition 6** *Even when there is incomplete information, the first-best efficient solution may be implemented in some circumstances.*

For a simple example, suppose there are only two types of projects, H type and L type, with  $p_H = 1$ . When a project is perfectly feasible, the solution is to exert a constant level of effort until the project is completed. The optimal level of effort is set such that:

$$h(\psi'y - \psi) = r(-1 - \psi' + hR)$$

So long as  $-\psi + hysR > 0$ , it is individually rational for H-type projects to accept the above funding. If  $p_L$  is low enough that  $\frac{p}{(1-p)} < \frac{\psi}{-\psi+hysR}$ , then an L-type project would receive negative expected payoffs were they to pretend to be H-types. Thus, they would rather admit to being L-types, even if the FBO effort level for L-types is 0. If indeed  $p_L < \frac{1}{hR}$ , then the FBO effort level is zero for all time, and then the H-types prefer their own FBO allocation provided it is individually rational. ■

As expected payoffs vary continuously with project feasibility under the FBO solution, we know that the two feasibility types need not be as far apart as in the example, but because the FBO and the FFD effort paths are not identical, we know that if feasibilities are too close together, then firms will have an incentive to misrepresent their type were the grant authority using the FBO effort paths. This is obviously true if there is a continuum of feasibility types.

The question then arises, what happens when the FBO allocation is not incentive compatible; when project types would prefer a different available FBO allocation to their own.

The incentive compatibility conditions require that for any two project feasibilities,  $p_i$  and  $p_j$  the Second-Best Optimum (SBO) allocations must satisfy:

$$\int_0^\infty ((1-p_i)(-\psi_{it} + \psi_{jt}) + p_i((-\psi_{it} + hy_{it}R)e^{-A_{it}} - (-\psi_{jt} + hy_{jt}R)e^{-A_{jt}}))e^{-rt} dt \geq 0 \quad (14)$$

In order for the above to hold for the above case AND its symmetric case (where the  $i$ 's and  $j$ 's are switched), the following must be true.

**Proposition 7** *For any two projects, L and H, with H more feasible than L, the Second-Best Optimal present value of effort given infeasibility must be higher for the H type than the L given infeasibility, but the expected private returns given feasibility must also be higher for the H type.*

If we define the present value of effort cost given infeasibility for firm  $i$ ,  $\int_0^\infty \psi_{it}e^{-rt} dt$ , as  $C_i$  and the present value of potential benefits given feasibility,  $\int_0^\infty (-\psi_{it} + hy_{it}R)e^{-A_{it}}e^{-rt} dt$ , as  $B_i$ . From equation (14), above, we have that  $-(1-p_i)(C_i - C_j) + p_i(B_i - B_j) \geq 0 \geq -(1-p_j)(C_i - C_j) + p_j(B_i - B_j)$ . If  $p_i > p_j$ , then  $C_i - C_j > 0$  and  $B_i - B_j > 0$ . ■

In the case where the FBO allocation of effort is not incentive compatible, and therefore not SBO, one of the two inequalities will become binding and at least one of the SBO effort paths will not be the same as the FBO path.

When  $p_i$  and  $p_j$  are low enough that FFD effort is higher than FBO effort for both type  $i$  and  $j$ , then the project with the lower feasibility will have a binding incentive compatibility constraint. This is because FBO effort is increasing with feasibility, meaning that firms prefer the FBO efforts of (slightly) higher types to their own.

This analysis can be extended to the case where we have a continuum of types. Assuming that the density of types supports a fully-separating equilibrium, all types must prefer their own effort path to that of any other type. Using the notation above, we have that  $-(1-p)C'(p) + pB'(p) = 0 \forall p$ , with  $C'(p) > 0$  and  $B'(p) > 0$ . As with the two type case, more feasible projects both have higher benefits if the project is feasible and higher costs if it is not.

This proposition has direct implications for a practical grant authority. It shows that firms that are more certain in the feasibility of their project should be willing to endure a quicker pace of research and slightly less comfortable conditions than they would prefer. This is to ensure that lower feasibility firms will remain content with a slower pace of work that is less likely to lead to a marketable innovation.

## 4 Discussion and Future Research

The current paper looks at a model where projects meant to achieve large social benefits are uncertain in their ability to generate a successful innovation. Even those projects that are feasible are uncertain in the timing of their innovation. Because of this, firms learn to update their beliefs about the feasibility of their project as time passes.

Due to the large potential social gains, the benevolent grant authority issues grants to innovating firms to encourage larger research efforts. While first-best effort paths also depend only on the current feasibility of a project, the grant authority must alter the effort paths to properly incentivize firms to truthfully reveal the feasibility of their projects. Such alterations are believed to lead to quicker initial paces of research for more feasible projects, but with sharp drop-offs in funding. Less feasible firms, meanwhile, get lower but more stable funding.

The current model is meant to be a useful baseline from which to compare other variations. Obvious extensions include letting several firms bid (or compete) for the right to a grant. Similarly, the grant authority may have the capability to issue several grants for

related projects, and it would be interesting to see the degree to which a grant authority would want to ‘hedge their bets’ by funding two or more projects in case one or more turn out to be infeasible.

The current paper also assumes that the grant authority has no information about the feasibility of a given grant. In reality, grant authorities employ experts that must evaluate each grant application, and are likely to reject project proposals if the referee’s estimate of a project’s feasibility is far outside the firm’s estimate. This process would likely reduce the information rents given to proposing firms.

Another major extension that would be desirable would be one where firms and the government may have different and/or evolving estimates of the potential returns (both social and private) that a given innovation may generate. Furthermore, the current analysis assumes that while firms differ in their feasibilities, they all have the potential to create the same quality of innovation. In reality, more able firms may not only have a better chance for successful innovation, but are likely to create a valuable product.

Finally, the current paper explores only the case where all social and private benefit is generated from the invention of the intended product. Innovate efforts, whether successful or not, typically lead to increased general knowledge about a field of study and can lead to other, seemingly unrelated, innovations in the future. Whether or not a model that incorporated a value to general human capital development would change the results significantly has yet to be seen.

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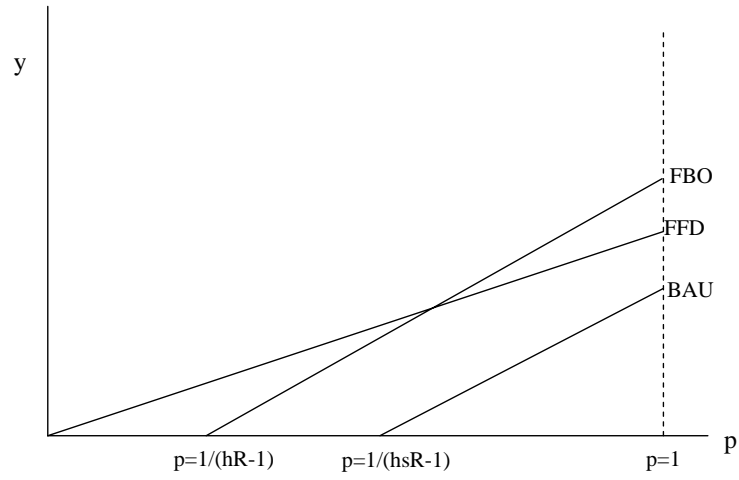


Figure 1: Optimal Effort by Feasibility and Path