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INTEGRATING DEMAND-MANAGEMENT WITH DEVELOPMENT OF SUPPLY-SIDE SUBSTITUTES

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Chapter 4: Integrating demand-management with development of supply-side substitutes

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Abstract

Sustaining water availability at current prices in the face of growing demand and declining resources is not possible, and scarcity is further exacerbated by falling recharge levels due to climate change, urbanization, and watershed depreciation. We discuss an integrated approach to water-resource development based on principles of sustainability science. In addition to demand-management such as pricing, we consider supply-side substitutes such as desalination and wastewater recycling. The importance of integrating demand- and supply-side approaches is especially evident in the case of watershed conservation as climate adaptation. Watershed conservation reduces scarcity by improving groundwater recharge. Yet, incorrect pricing can waste those potential gains. We discuss a joint management strategy, wherein block prices for groundwater consumption and co-determined prices for watershed conservation incentivize and finance efficient profiles of both.

1 Introduction

Freshwater scarcity has long been an important issue in many regions across the globe. The United Nations recommends a multidisciplinary approach to management, inasmuch as water scarcity "affects all social and economic sectors and threatens the sustainability of the natural resource base" (United Nations, 2006). While the scope of the problem is clear, more research is needed to effectively integrate demand- and supply-side water management strategies in a systematic manner. To that end, we develop and discuss a framework for assessing and prioritizing a variety of water management instruments.

Sustaining water availability at current prices in the face of growing demand and declining resources is not possible, especially when taking into account the effects of climate change, urbanization, and watershed degradation. Implementation of costly ground or surface water alternatives, such as desalinated seawater or recycled wastewater, can be delayed, however, with appropriate long-run planning. For example, investing in watershed conservation reduces scarcity by improving groundwater recharge. Yet, incorrect groundwater pricing can waste those potential gains. Relatedly, payments for ecosystem services such as recharge are likely to be mispriced unless system interdependence is properly accounted for.

In this chapter, we discuss the joint implementation of three management tools: (i) water pricing, (ii) watershed conservation, and (iii) wastewater recycling. We then investigate the possibility of extending the framework to include multiple groundwater resources and allowing for the effects of climate change explicitly.

2 Watershed conservation

We begin by examining how watershed conservation activities, integrated into a standard groundwater management framework, affect optimal outcomes for head levels (an index of the groundwater stock), extraction, and present value. Unlike alternative water sources which act to supplement groundwater extraction, watershed conservation enhances existing sources by increasing the proportion of precipitation that reaches subterraneous aquifers as recharge. There are many types of conservation activities – e.g. reforestation, removal of feral ungulates, removal of invasive plant species, construction of settlement ponds or injection wells – but the purpose of this chapter is not to develop a methodology for optimizing across a spectrum of potential conservation instruments.[1] Rather, assuming available instruments are employed in an optimal manner, we examine the interrelated nature of groundwater conservation (reduced extraction) and investment in watershed conservation capital. Both types of conservation are steered by the shadow price or scarcity value of groundwater, and both instruments should be employed until their marginal benefits equal their marginal costs.

The groundwater resource is modeled as a single-cell coastal aquifer (Krulce et al. 1997), such that the volume of stored water is directly proportional to the head level (h) or the vertical distance from mean sea level to the water table. The head level changes over time according to recharge (R), stock-dependent natural leakage to adjacent water bodies (L), and extraction (q):

$$\dot{h}_t = R(N_t) - L(h_t) - q_t \tag{1}$$

where $R' \geq 0$ and $R'' \leq 0$. Groundwater recharge is an increasing function of the conservation capital stock (N), but the effectiveness of additional investment declines as the stock of capital increases. The decreasing marginal productivity of N captures the idea that a dynamically

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[1] See chapter 12 in this volume for a framework that allows for different types of conservation instruments.

optimized portfolio of conservation activities would employ the most cost-effective instruments first. Thus depending on the characteristics of the resources, it may make sense to never use one or more of the available watershed conservation instruments.

Produced conservation capital naturally degrades over time (e.g. a fence), and requires maintenance. Investment in removal and replacement of natural capital (e.g. replacing invasive weeds with native forest) also requires maintenance – invasive plant seeds can remain viable for many years. Generally, investment (I) in conservation activities at unit cost c_I can be made to both offset the natural rate of capital depreciation (δ) and to build up the existing stock:

$$\dot{N}_t = I_t - \delta N_t \tag{2}$$

The constant unit cost of investment could instead be modeled as an increasing and convex function of N to capture the idea that the lowest-cost areas are targeted first; in the case of fencing, this might correspond to areas with the easiest terrain to traverse. However, the concavity of the recharge function already implies optimization across available conservation instruments. Either specification is sufficient to illustrate that the marginal net benefit of conservation varies with the stock of capital.

The management problem is to choose the rates of groundwater extraction (q), an alternative backstop resource such as desalinated brackish water (b), and investment in conservation capital (I), given a non-negative discount rate (r) to maximize net present value:

$$\max_{q_t, b_t, I_t} \int_{t=0}^{\infty} e^{-rt} \left\{ \int_{x=0}^{q_t+b_t} D^{-1}(x,t)dx - c_q(h_t)q_t - c_b b_t - c_I I_t \right\} dt \tag{3}$$

subject to equations (1) and (2), non-negativity constraints on the control variables, and $h_t \geq h_{\min}$, where the minimum head constraint is determined, for example, by the minimum allowable salinity for potable water. The period-t benefit is measured as consumer surplus, or the

area under the demand curve up to the optimal quantity. Defining the marginal benefit along the optimal trajectory or *efficiency price* as $p_t \equiv D^{-1}(q_t + b_t, t)$, one can derive a dynamic pricing equation that is driven by the head level and the capital stock (Roumasset and Wada, 2013):

$$p_t = c_q(h_t) + \frac{\dot{p}_t - c'(h_t)[R(N_t) - L(h_t)]}{r + L'(h_t)} \tag{4}$$

To incentivize extraction of the optimum quantity, the price should be set equal to the sum of the marginal extraction cost and the marginal user cost (MUC), the latter of which represents the loss in present value resulting from extracting a single unit of the resource at the time of extraction.

An equimarginality condition can also be derived to guide optimal investment in conservation capital (Roumasset and Wada, 2013):

$$\lambda_t = \frac{c_I(r + \delta)}{R'(N_t)} \tag{5}$$

where λ is the shadow price of groundwater or the costate variable associated with the dynamic state equation for the head level (equation 1). The numerator on the right hand side of equation (5) is the marginal opportunity cost of investment or the user cost of capital (Jorgenson, 1963), which includes the forgone interest that would have accrued had the income not been invested in capital and the cost of depreciation. The denominator is the marginal product of capital in recharge and converts the user cost to dollars per unit of recharge. The entire expression can be viewed as a supply curve for recharge, inasmuch as the diminishing marginal productivity of capital implies an increasing function in N. The left hand side of equation (5) is the marginal benefit of recharge via conservation capital, measured as the shadow price of groundwater. Welfare is maximized where the recharge supply curve intersects the shadow price of groundwater, which is also equal to the net marginal benefit of groundwater extraction along the

optimal path. In summary, the resource manager should be indifferent between conserving

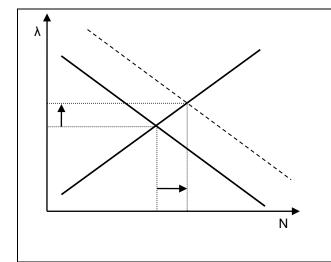groundwater via consumption reduction and via investment in the watershed.

Figure 1. Demand and Supply of Recharge

The marginal benefits of recharge are given by the marginal opportunity cost of groundwater (the sum of marginal extraction cost plus marginal user cost), which is increasing over time with groundwater depletion. Consequently optimal natural capital, N, and the shadow price of groundwater, λ, increase over time toward their steady state levels as investment is ramped up.

Assuming the existence of a steady state, wherein $\dot{p} = \dot{h} = \dot{N} = 0$, the optimal investment rule

is to choose the maximum feasible level of investment in every period prior. The evolution of the

capital stock, in turn, is determined by equation (2), and the optimal head and extraction paths

are steered by equations (1) and (4) respectively. When demand is increasing and unbounded, the

price of water eventually rises to the backstop price. Moreover, because extraction costs typically

do not increase substantially as the head level is drawn down, the optimal steady state likely

entails depleting the aquifer to its minimum head level to minimize leakage. In general, near

constant extraction costs result in a shorter time to steady state, whereas rapidly increasing

(convex) costs tend to lengthen the transition to the steady state.

Because the current value Hamiltonian corresponding to equation (3) is linear in investment,

the dynamic paths of capital stock and investment will approach monotonically from above or

below the steady state target, depending on the initial value N_0. The Hamiltonian is not linear in

groundwater extraction, however, and the optimal trajectories of the groundwater stock,

efficiency price, and extraction may take a non-monotonic approach to the steady state. In the case of constant aquifer recharge, it has been shown that a period of aquifer replenishment may optimally precede drawdown in anticipation of future scarcity (Krulce et al., 1997; Roumasset and Wada, 2012).

3 Water recycling

While watershed conservation enhances existing groundwater sources, recycled water serves as a groundwater alternative, much like desalinated brackish or sea water. If treatment costs (inclusive of amortized capital) were identical, there would be little reason to prefer one alternative to the other, aside from environmental considerations – sludge left over from both treatment processes need to be deposited somewhere. However, desalinated water is generally treated to potable standards, making it a perfect substitute for groundwater, whereas treating wastewater to potable standards for reuse is often not cost effective. As an imperfect substitute, recycled water may optimally supply only a subset of water users, whose marginal benefit of lower quality water is still high enough to justify treatment costs. In the simplest case, and the one discussed in this section, recycled water provides the same marginal benefit as groundwater for a subset of demand sectors and does not meet quality standards for the remaining sectors. Recycled water, therefore, serves as a sector-specific supplemental resource that facilitates substitution of groundwater usage toward sectors that require high quality water.

We consider the problem of allocating three types of water – groundwater, recycled water, desalinated water – to two demand sectors. The household sector (H) can only use potable water provided by the aquifer or the desalination facility, whereas the agriculture sector (A) can draw from any of the three sources. Because recycled water is assumed to be of lower quality, a

separate distribution infrastructure is required. Infrastructure investment can be accounted for by assuming a convex marginal cost of delivering recycled water, i.e. $c_R(q_t^R) > 0$, $c_R'(q_t^R) > 0$, and $c_R''(q_t^R) \geq 0$. Implicitly, treatment facilities are initially constructed near the densest area of users, and the distribution network endogenously expands over time as the proportion of users in the agriculture sector increases in accordance with rising groundwater scarcity. The groundwater hydrology is described by equation (1), with a slight modification; the quantity of groundwater extracted (q) must be further disaggregated into extraction for the household (q^{GH}) and agriculture (q^{GA}) sectors. The management problem is to choose groundwater extraction for each sector, desalination for the household (q^{BH}) and agriculture (q^{BA}) sectors, and water recycling for the agriculture sector (q^{RA}) to maximize net present value subject to the modified aquifer equation of motion, non-negativity constraints on the control variables, and a minimum head level constraint.[2] The marginal cost of a particular resource is the same between sectors – with the exception of recycled water whose marginal cost is implicitly infinite for the household sector – but the marginal benefit varies according to the sector-specific demand functions D_H and D_A. One can show that efficient water use in each sector is achieved by equating the marginal benefit with the lowest available *marginal opportunity cost* (Roumasset and Wada, 2011):[3]
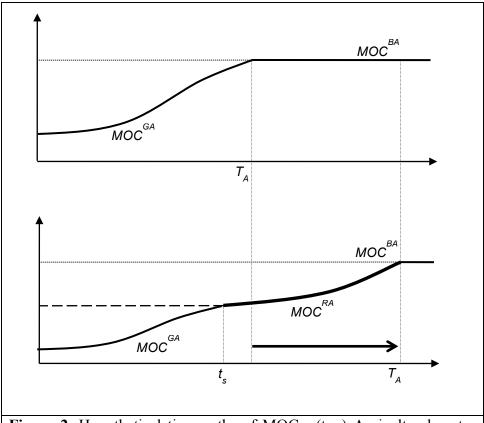
$$p_t^H = \min\{c_G(h_t) + \lambda_t, c_B\} \tag{6}$$

$$p_t^A = \min\{c_G(h_t) + \lambda_t, c_B, c_R(q_t^{RA}) + q_t^{RA}c_R'\} \tag{7}$$

In general, a particular resource need not be used in both sectors or at all. For example, if the scarcity value of groundwater is initially low, both demands are supplied by groundwater. As

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[2] See the appendix for a rigorous mathematical representation of the management problem.

[3] See chapter 3 in this volume for a more detailed discussion of the marginal opportunity cost and optimal ordering of water resources.

scarcity rises, it becomes cost-effective to supplement groundwater extraction with recycled water in the agriculture sector, while the household sector continues to use groundwater exclusively. In the steady state, all available resources are used in each sector.

Figure 2. Hypothetical time paths of MOCs: (top) Agricultural sector without recycling, (bottom) Agricultural sector with recycling.

The gains from recycling are most apparent in the sectors that are able to use recycled water. In Figure 2, the agriculture sector postpones the use of costly desalination from T_A in the top panel to T_A in the bottom panel. Starting from period t_s, recycled water is used to supplement groundwater, and users enjoy lower prices over a longer period of time. Households also gain from the recycling program because substituting groundwater in the agriculture sector means that scarcity is reduced for *all* groundwater users. Although the optimal MOC is rising over time, total consumption may be rising or falling, since the growth in demand and the price effect work

in opposite directions. If the effects are of equal magnitude or are both relatively small, optimal consumption may be nearly constant.

4   Systems approach to water management

Fully characterizing tradeoffs across sources and end uses requires integration of all available demand- and supply-side management strategies. In the examples discussed in this chapter, demand-side management included sector-specific pricing, and supply-side management included two types of instruments: (i) augmentation of existing groundwater resources (watershed conservation) and (ii) optimal implementation of groundwater alternatives (water recycling and desalination). With multiple demands, resources, and management instruments, the framework should more explicitly account for spatial heterogeneity.

Consider the case of two consumption districts and two aquifers, where each of the aquifers has its own watershed, each of the consumption districts has its own desalination and recycling substitutes, and the transport costs of water from one district to the other are given.[4] Assume moreover that if no water is transported from one district to another that each of the consumption districts is more cheaply served by the aquifer nearest to it. In this case one simply solves for each watershed-aquifer-district separately by simultaneously satisfying the conditions from section 2 and 3 for each system. However for a fully internal solution with positive transport from one system to another, the two systems themselves must be simultaneously solved such that the shadow price of the recipient district is equal to the shadow price of the source district plus transport costs. In this case there is effectively one demand, the aggregation of the source

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[4] The framework can be further generalized by incorporating additional distribution costs within a particular district, e.g. for pumping water to users located at different elevations (Pitafi and Roumasset, 2009).

demand and the recipient demand shifted up by the amount of the transport costs (Pitafi and Roumasset, 2009). Accordingly, the order of extraction from the two aquifers can be solved by the principle of least-MOC-first. In the case of "leaky" coastal aquifers, this may mean that the largest and leakiest aquifer should be used first since the opportunity costs of future use is offset by the reduction of leakage (chapter 3 in this volume). In the case of inland aquifers with nearly constant extraction costs, it will typically mean the aquifer with the lowest extraction cost is used first.

5   Implementation

Assuming that production of recycled water is never constrained by the availability of primary feed water, its marginal opportunity cost is captured entirely by the marginal cost of treatment, inclusive of construction costs for the treatment facilities and distribution infrastructure. Thus, implementation is relatively straightforward. In the case of watershed conservation, however, the marginal opportunity cost of groundwater depends on upstream decisions to invest in natural capital. Inasmuch as the upstream providers of groundwater recharge services generally receive only a portion of the benefits generated by their investment in natural capital, welfare maximization for society as a whole often requires some means of compensating upstream landowners, i.e. a *payment for watershed services* (PWS) plan.

The PWS approach falls under the broader umbrella of *payments for ecosystem services* (PES), which has advanced as a method for incentivizing provision of any number of ecosystem services by private landowners (Daily et al., 2009). PES, which are "voluntary transactions where a well-defined environmental service is being bought by a service buyer from a service provider if and only if the service provider secures service provision" (Engel et al., 2008), are
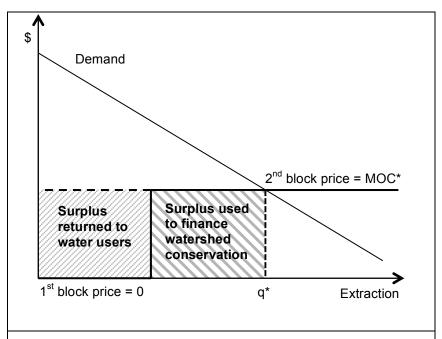
sometimes set in accordance with the perceived opportunity cost of upstream landowners (e.g. Muñoz-Piña et al., 2008). In other cases, ecosystem services prices are viewed as exogenous, meaning private providers can be theoretically incentivized by payments according to a fixed Pigouvian subsidy (Baumol and Oates, 1988). If the ecosystem service in question provides indirect benefits via another natural resource, however, the marginal benefit of the service is only revealed as part of a joint solution (Sathirathai and Barbier, 2001; Barbier et al., 2002; Barbier, 2007; Sanchirico and Springborn, 2011; Roumasset and Wada, 2013). For example, the value of groundwater recharge provided by a watershed depends on both the quantity of groundwater extraction and the amount of watershed conservation.

While most researchers appear to be in agreement that beneficiaries should at least be partially responsible for financing payments, the economic foundations underlying many PES programs remain largely informal and leave open important questions, such as how payments should be precisely determined and how conservation investment should be financed without distorting incentives. In practice, a common way to finance payments for ecosystem services is to use general revenue (Liu et al., 2008; Pagiola, 2008), which has the disadvantage of increasing required tax revenues and consequently generating additional marginal excess burden (Ballard et al., 1985). A frequently employed alternative to financing from general revenue, especially for PWS programs, is a volumetric tax – e.g. Houston's (Texas) dedicated conservation fees, Salt Lake City's (Utah) Watershed-Water Rights Purchase Fund Program, Rhode Island's Watershed Land Acquisition Program, and New York City's Catskills Watershed Management Plan. Although these types of programs target beneficiaries in the spirit of PES/PWS and generate revenue, they are not welfare maximizing because user fees drive a wedge between marginal

benefits and costs. One way to avoid this problem is to finance PES through intergenerational benefit taxation.

Optimality conditions (4) and (5) confirm that the marginal benefit of the upstream watershed service is dependent on the shadow price of the downstream groundwater resource. Private producers of the recharge service would only invest in watershed conservation if the payments received from doing so are at least equal to the costs, including the opportunity cost of the land. In the simplest case in which the opportunity cost of the land is negligible, e.g. in upper watershed areas not particularly suitable for urban or agricultural development, Roumasset and Wada (2013) show that the decentralized solution for ecosystem service provision emulates the social optimum if the government sets the payment for each unit of recharge equal to the shadow price of groundwater, adjusted for the discount rate and capital depreciation, and water is priced at its marginal opportunity cost. Optimal ecosystem service and water pricing generates a revenue surplus, which can be returned to water consumers in lump sum fashion with appropriate block pricing (e.g. Pitafi and Roumasset, 2009). Because the water price need only be correct at the margin, the size of the first price block can be adjusted to return surplus to water users through free inframarginal units of water.

Figure 3. An increasing block pricing structure finances
conservation and returns revenue surplus to water consumers.
For clarity of exposition, we assume zero extraction costs.

In some instances, the size of a conservation project is determined exogenously by

policymakers or the conservation instrument may exhibit non-convexities that render marginal-

product pricing infeasible. A large public project may incur substantial costs initially, even

though most of the benefits are generated in future periods. If the revenue surplus is not

sufficient to cover initial investment, bond-financing will be required, such that the present value

of collections is equal to the present value of investment. A lump sum proportional benefit tax

can preserve efficient incentives while maintaining a balanced intergenerational budget. Under

such a program, each generation pays a fixed proportion of its watershed conservation induced

benefits – lower water prices and delayed implementation of expensive alternatives such as

desalination – to pay off the bond required to finance the conservation project.

6 Adapting to climate change

Existing climate models, although highly uncertain, predict that the frequency of storm events will increase in many regions across the globe and mean rainfall will decrease in some regions in the coming centuries. If the projections turn out to be reasonably accurate, the net effect will be an increase in runoff and a decrease in recharge to groundwater resources in many areas. Adapting to climate change, therefore, requires modifying groundwater management frameworks to allow natural recharge to decline over time. The equimarginality conditions for groundwater extraction, recycled water use, and watershed conservation remain unchanged, except that recharge is a function of both conservation capital and a time-varying component that captures the effect of climate change. The general effect of optimally employing various conservation instruments (compared to managing only extraction) is the same with or without climate change: scarcity is reduced and the resource is drawn down more slowly toward its steady state level. However, patterns of extraction and investment in conservation capital may be substantially different in transition to the steady state.

As previously discussed, a period of aquifer replenishment may optimally precede drawdown in the case of constant recharge. That is to say the head level may follow a non-monotonic path to the steady state. The incentive to build the groundwater stock may be even larger in the face of climate change, given that future water scarcity is expected to be higher owing to declining recharge. Similarly, optimal watershed conservation will initially increase in the short and medium term. But since the unit cost of supplying recharge via watershed conservation increases as rainfall declines, the groundwater scarcity effect may be eventually dominated, resulting in a non-monotonic path of optimal watershed investment.

7   Challenges and research needs

While many advances have been made in the theory of integrated water resource management, operationalizing optimization models over time and space remains a challenge. One particularly daunting challenge involves interfacing complicated physical watershed and groundwater models with an economic optimization framework. Starting with a simple single-cell aquifer, one can imagine a grid representing the watershed landscape. Assuming satellite or other spatially disaggregated data is available, one could characterize each section of the grid in terms of slope and permeability, the latter being a function of investment in natural capital and/or physical capital. The first problem is to determine the optimal allocation of investment across the grid, such that the marginal dollar invested in each cell yields the same recharge. The next step is to implement the watershed optimization model, increasing total investment in natural capital until the marginal opportunity cost equals the shadow price of water, co-determined with aquifer management. Assuming that the first stage of the optimization problem – allocating investment across the grid – is feasible, the remainder of the problem can be solved using standard methods, given that we need only keep track of two state variables (conservation capital and groundwater).

The problem becomes more complicated when the hydrology is instead described by a higher dimensional physical model, wherein the aquifer is divided into multiple cells with water moving between cells over time. Even if one could establish a method for optimizing pumping across available wells in every period to satisfy the total optimal rate of extraction, both the head levels and recharge will vary across cells, and cell-specific head levels will depend on disaggregated inflows to and outflows from the aquifer. And while complicated spatial hydrology can be modeled using numerical methods, optimization solutions become increasingly less reliable and eventually unattainable as the number of state variables – in this case head

levels associated with each cell – grows. The management problem becomes even more complex when climate change is considered, e.g. Monte Carlo methods may be required to generate rainfall sequences that serve as inputs to the watershed model.

One way to address these research challenges is to improve search algorithms for numerical solutions or to find and apply existing algorithms previously unused in the field of resource economics. Another approach is to refine the simplifying assumptions underlying tractable lower-dimensional models that are currently solvable with existing algorithms. Refinements might include, for example, improving spatial aggregation methods or simplifying linkages between various model components. The two approaches are not mutually exclusive, however. The validity of proposed model refinements can and should be tested to the extent feasible using existing numerical models.

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9   Appendix

The groundwater hydrology for the 2-sector 3-resource management problem is described
by the following equation:

$$\dot{h}_t = R(N_t) - L(h_t) - (q_t^{GH} + q_t^{GA}) \tag{A1}$$

where the total quantity of groundwater extracted in every period is equal to the sum of
household extraction (q^{GH}) and agricultural extraction (q^{GA}). The marginal benefit of a unit of
water is represented by sector-specific demand functions D_H and D_A for the household and
agriculture sectors respectively. The management problem is to choose groundwater extraction
for each sector, desalination for the household (q^{BH}) and agriculture (q^{BA}) sectors, and water
recycling for the agriculture sector (q^{RA}) to maximize net present value:

$$\max_{q_t^{GH}, q_t^{BH}, q_t^{GA}, q_t^{RA}, q_t^{BA}} \int_0^{\infty} e^{-rt} \left\{ \begin{array}{l} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)dx + \int_0^{q_t^{GA}+q_t^{RA}+q_t^{BA}} D_A^{-1}(x,t)dx - \\ (q_t^{GH}+q_t^{GA})c_G(h_t) - (q_t^{BH}+q_t^{BA})c_B - q_t^{RA}c_R(q_t^{RA}) \end{array} \right\} dt \tag{A2}$$

subject to equation (A1), non-negativity constraints on the control variables, and a minimum
head level constraint.