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Intergenerational Equity with Individual Impatience in an OLG Model of Optimal and Sustainable Growth

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Abstract

Among the ethical objections to intergenerational impartiality is the violation of consumer sovereignty given that individuals are impatient. We accommodate that concern by distinguishing intra- and inter-generational discounting in an OLG model suitable for analyzing sustainability issues. Under the assumption of constant elasticity of marginal felicity, the optimum trajectory of aggregate consumption is guided, via the Ramsey condition, by the intergenerational discount rate but not the personal discount rate. In an economy with produced capital and a renewable resource, *intergenerational neutrality* results in a sustained growth path, without the necessity of a sustainability constraint, even in the presence of intragenerational impatience. We also find that green net national product remains constant along the optimal approach path to golden rule consumption.

Key words: Sustainability of optimal growth, intergenerational equity, intra-generational discounting, renewable resources, GNNP

JEL Codes: Q56, Q41, Q01

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If I am not for myself, then who is for me? If I am not for others, then who am I?

Rabbi Hillel[1]

1. Introduction

The debate surrounding discounting, especially as it applies to the prospect of sustainable growth, has only intensified in the context of climate change (Stern, 2006; Nordhaus, 2007; Weitzman, 2007; Sterner and Persson, 2008; Heal, 2009). Intergenerational equity, and sustainability more generally,[2] are inherently normative inquiries (Solow, 2005; 2006). Nonetheless, social welfare functions are typically assumed to embody consumer sovereignty, including time preferences.[3] But how can we reconcile social preferences that respect *intergenerational neutrality*[4] and still base the social welfare function on individual lifetime

[1] As quoted by Arrow (1974, p. 15).

[2] While there are many different definitions, "sustainability" in this paper is taken as founded on the *three pillars* of intergenerational equity, interlinkages between the environment and the economy, and dynamic optimization (Stavins et al., 2003).

[3] As noted by Sandmo (1983), "The Bergson-Samuelson social welfare function is usually taken to reflect the principle of consumer sovereignty; social welfare evaluations should respect individual preferences."

[4] For reasons detailed in section 2, we believe that "intergenerational neutrality" (Koopmans 1965) is a compelling answer to Solow's (1986) challenge: "How much of the world's – or a country's – endowment of a nonrenewable resources is it fair for the current generation to use up, and how much should be left for generations to come who

utilities and the personal rates of time preference that they embody? A way out of the dilemma is to differentiate intragenerational impatience from the planner's utility discount rate.[5]

In order to engage the sustainable growth literature, we develop a continuous time overlapping generations model (Yaari, 1965; Blanchard, 1985; Blanchard and Fisher, 1989; Calvo and Obstfeld, 1988) with produced capital as well as natural resources, which incorporates the distinction between inter- and intra-generational utility discounting. While others have shown that intergenerationally neutral and optimal economic development is sustainable in the presence of renewability and/or a backstop (Heal, 2000; Ayong Le Kama, 2001; Endress et al., 2005),[6] the question of how to include individual impatience remains unaddressed. A key result of this paper is that incorporating information about intragenerational impatience into the model does not affect the level of aggregate consumption, which is governed solely by the intergenerational discount rate, ρ. Consequently, the conditions required to implement Koopmans' (1965) method of imposing intergenerational neutrality ($\rho = 0$) are satisfied.[7] We find that the optimal trajectory

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have no active voice in contemporary decisions?" Of course other ethical positions are possible (see e.g. Dasgupta, 2011).

[5] Arrow (1999) and Dasgupta (2001) criticize zero utility discounting, albeit without discussing this distinction (see section 2).

[6] Valente (2005) provides conditions for optimal growth to be sustainable in a model with positive utility discounting, resource renewability, and technical change.

[7]Koopmans (1965) has not enjoyed the prominence in the growth literature that Koopmans (1960) and Koopmans et al. (1964) have. Perhaps this is due to its original publication in a volume of essays rather than in a prominent journal. Nonetheless, Koopmans (1965) presents a way out of the dilemma of "timing neutrality" identified in the earlier papers. An alternative approach, based on the overtaking criterion, is presented in Weizacker (1965), wherein Weizacker proves a theorem about sufficient conditions for the existence of an optimal path. The theorem is then shown to be a generalization of Koopmans' (1965) theory of optimal growth in an economy with population growth,

of aggregate consumption is rising, which implies that a sustainability constraint, if imposed, would be non-binding in the model.[8] Relatedly, sustainable income, defined as Green Net National Product (GNNP), is actually sustained along the optimal path, avoiding a paradox in the conventional approach.

Our focus is on sustainability and intergenerational equity, which distinguishes the present paper from others that also employ the OLG framework, some involving a two-stage optimization process that we adapt and apply. Calvo and Obstfeld (1988) analyze aspects of optimal fiscal policy for economies with capital accumulation and finitely-lived, heterogeneous agents. For a particular utilitarian social welfare function, the problem faced by a central planner is broken down into two subproblems, a standard problem of optimally allocating aggregate consumption over time and a problem of distributing aggregate consumption optimally at each moment among those alive. But the application is to fiscal policy and issues of time consistency, not sustainability, which is the theme of our paper.

Motivated by the report of the World Commission on Environment and Development (1987), Howarth and Norgaard (1992) use a discrete time OLG model to examine sustainability and intergenerational fairness, primarily in the context of energy use, a stock pollutant, and climate change. In the spirit of the Second Theorem of Welfare Economics, Howarth and Norgaard (1992) incorporate income transfers that lead to Pareto efficient outcomes under

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no technical progress and no (i.e. zero) time preference. Gale (1970) extends this analysis to optimal development in a multisector economy. Becker and Boyd (1997) and Dana and Le Van (2006) present modern versions of Weizacker (1965) overtaking. So called "good programs," based on Gale (1967) and analogous to Koopmans' (1965) "eligible paths," approach golden rule utility.

[8] See Endress et al. (2005) for a derivation of rising consumption with intergenerational neutrality albeit without intragenerational impatience.

competitive general equilibrium that exhibit greater intergeneration fairness, though not necessarily intergenerational neutrality. However, Howarth and Norgaard (1992) do not address the problem of consumer sovereignty, nor do they distinguish between inter- and intra-generational discount rates.

Using a constant-population, cake-eating model without capital, Burton (1993) shows that inter- and intra- generational discount rates affect optimal resource harvesting decisions in different ways. Howarth (1998), Marini and Scaramozzino (1995), and Copeland and Taylor (2009) have built on Burton's model regarding optimal and equilibrium issues of resource management. Our focus is to develop a model capable of incorporating neutral weighting across generations without negating individual impatience.

Schneider et al. (2012) focus on intergenerational tradeoffs involved in climate change mitigation, arguing that continuous time OLG models are more appropriate for studying relevant tradeoffs than standard infinitely lived agent (ILA) models, which deliver the Ramsey equation. Various propositions establish conditions under which both decentralized and centralized OLG models are observationally equivalent to ILA models. The paper goes through a two stage OLG optimization similar to Calvo and Obstfeld (1988); the result is collapse to an equivalent ILA model, wherein individual time preference drops out.

Our paper presents a more general framework by including a renewable resource X, which allows for the derivation of a Hotelling first order condition, a result not discussed by Schneider et al. (2012). We also address sustainability more broadly, not just the tradeoffs involved in climate change mitigation. Our results have major implications for Green Net National Product (GNNP), which Schneider et al. (2012) do not address. In particular, our formulation (with intergenerational equity, even with individual impatience) promotes sustainability without ad hoc constraints, which would otherwise distort shadow prices fundamental to the computation of GNNP.

The result that intergenerational neutrality could be achieved without violating individual consumer sovereignty is not surprising if one thinks of the works of Ramsey (1928), Solow (2005, 2006), Koopmans (1965), Burton (1993), Blanchard (1985), Calvo and Obstfeld (1988), and Endress et al. (2005), together. In the spirit of Weitzman (1998),[9] our contribution lies in drawing on the disparate literature to develop a suitable framework for our question and deriving the (possibly expected) results, inasmuch as none of the individual models is sufficiently general to imply the results in question. We believe that the results are of consequence because they reconcile the perceived conflict between intergenerational equity and individual impatience in the economics of sustainable development.

Section 2 provides an overview of the debate around utility discounting, including objections to assuming a rate of zero. In section 3, we extend the neoclassical approach to sustainable growth to allow separate intragenerational and intergenerational discount rates. The social planner respects consumer sovereignty regarding lifetime utility, thus incorporating the individual's personal rate of time preference via intragenerational discounting. The intergenerational discount rate is based on the planner's preferences regarding intergenerational equity. In section 4, we consider the special case of intergenerational neutrality ($\rho = 0$). As an extension to our basic continuous time model, section 5 assigns amenity value to the resource X by including it in the utility function and treating it as non-rival, as Gerlagh and Keyser (2001) do. Our key results remain valid in this extended model: intergenerational neutrality, a key

[9] "Production of new ideas is made a function of newly reconfigured old ideas in the spirit of the way an agricultural research station develops improved plant varieties by cross-pollinating existing plant varieties." (Weitzman, 1998)

principle of sustainability, remains technically feasible and can be achieved without violating consumer sovereignty. We demonstrate that a policy instrument like an amenity tax can approximate the central planner's neutral weighting of generations ($\rho = 0$). Section 6 summarizes and concludes.

2. Discounting and intergenerational neutrality

Neoclassical theories of sustainable growth have predominantly utilized a positive utility discount rate. This is odd because sustainability has everything to do with intergenerational equity (e.g. Stavins et al., 2003; Solow, 2005), and intergenerational equity is often interpreted as requiring a zero rate of pure time preference. Ramsey (1928) is often cited for his forceful pronouncement that discounting is "ethically indefensible". Samuelson and Solow (1956) generalize the Ramsey (1928) model with zero discounting to any number of capital goods. Solow has opined from a Rawlsian impartial spectator perspective ("in solemn conclaves assembled," Solow, 1974 p. 9) that "we ought to act as if the social rate of time preference were zero."[10] Pigou (1920) stated that pure time preference implies that "our telescopic faculty is defective," and Harrod (1948) said that "pure time preference is a polite expression for the conquest of reason by passion."[11] Koopmans (1965, p. 239) expressed "an ethical preference for neutrality as between the welfare of different generations." And Heal (2009) has called a positive pure rate of time preference "the rate of intergenerational discrimination.[12] Moreover, the optimal growth trajectory with positive utility discounting in an economy with an initial

[10] The impartial spectator perspective does not imply Rawls's maximin or difference principle, which was only intended to apply to intragenerational equity.

[11] This and the Ramsey, Solow, Pigou, and Harrod quotes are noted by Arrow (1999, p14).

[12] Koopmans (1965) and Dasgupta (2008) refer to the rate time preference as "felicity discounting."

endowment of a non-renewable resource is unsustainable in the sense that the optimal consumption trajectory is eventually declining (Dasgupta and Heal, 1979). In a recent discussion on discounting, Dasgupta (2011) points out that "many moralists have expressed the same sentiment many times before, even insisting that the principle of awarding equal weights to the welfares of all generations trumps every other moral consideration (Sidgwick, 1907; Pigou, 1920; Ramsey 1928; Parfit, 1984; Cowen and Parfit, 1992; Broome, 2008)."[13] Despite these advantages of intergenerational neutrality, the assumption has been challenged on technical and moral grounds.

There are several objections to zero utility discounting that need to be dealt with at the outset. A technical difficulty with a utility discount rate of zero discussed by Heal (1993) and Dasgupta (2011) is the "cake-eating problem." In a simple cake-eating model of a finite resource, zero utility-discounting in such an economy implies that any arbitrarily-small initial consumption level is dominated by a still lower initial consumption level. This gloomy prospect is not a logical conundrum – indeed a complete ordering of all consumption paths exists. It is rather the dismal consequence of contemplating an economy wherein any positive level of consumption cannot be sustained. Advocating a positive discount rate in this case results in condemning future generations to consumption levels approaching zero – hardly consistent with intergenerational equity.

Arrow (1999) and Dasgupta (2001) argue that an individual rate of pure time preference would impoverish the present and emphasize the moral unacceptability of draconian savings in the first period. They show that in a Ramsey (1928) optimal growth framework with zero utility

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[13] While these sentiments are compelling, we are not insisting that ρ has to be zero. Rather we explore the implications of intergenerational impartiality regarding the nature and sustainability of optimal consumption paths.

discounting and an elasticity of marginal "felicity" of consumption, η, of 1.5 or less, the savings rate in generation one must be two thirds or more.[14] This would appear to discriminate unfairly against the present generation and violate the moral principle of *universalizability*.[15] There are two highly restrictive assumptions hidden in this calculation, however. One is that said elasticity has been estimated from revealed preference. As Arrow et al. have later acknowledged, however, the relevant elasticity concerns the *planner's* tolerance for intergenerational inequality, not that of an individual generation. The other problem is that intergenerational neutrality has been taken to imply zero utility discounting even within a generation, an assumption that we see below is unnecessary.

In setting the stage for an overlapping generations model of climate change, Howarth (1998) discusses the discounting dilemma, reviews arguments for and against zero utility discounting, and notes the conundrum of conflating empirical and normative foundations of discounting. "Since *is* does not imply *ought*, the derivation of normative precepts from empirical observations is logically problematic." Indeed this is the heart of the modern discounting debate in the context of mitigating climate change. As discussed in Dasgupta (2008) and Heal (2009), the Stern Review (Stern, 2006) invokes a pure rate time preference of almost zero (.1%) on ethical grounds. In contrast, Nordhaus (2007) allows that the discount rate should conform to

[14] For details of the calculation, see Dasgupta (2001), p. 255.

[15] While the definition of universalizability is contentious, one interpretation is the requirement that the maxim of your action be one that everyone could act upon in similar circumstances. Arrow cites the empirical studies in IPCC (1995) as justification. One difficulty in applying the universalizability criterion, however, is that the ethical grounds for the criterion, starting with Rabbi Hillel, derive from discussions of a finite number of individuals or income groups. As noted by Rawls (1971), they do not apply to the case of an infinite number of generations.

"marketplace real interest rates and savings rates" (p. 686),[16] and invokes the opportunity cost of capital interpretation of discounting.

Another argument arises when considering how different individuals interpret "fairness". It may appear, as Dasgupta (2011) notes, that setting ρ equal to zero confers an unfair advantage to future generations who have the "natural advantage" of larger stocks of capital knowledge and who can choose to accumulate stocks of renewable resources. But that advantage can be tempered by a high value of η, the elasticity of marginal felicity, which captures social tolerance for intergenerational equality. Indeed, Dasgupta argues elsewhere (Dasgupta, 2008) that η should be between 2 and 4. In no way does a high value of η dictate the value of ρ, a measure of (the lack of) intergenerational neutrality.

In a recent critique of intergenerational discounting, as discussed for example in Dasgupta (2005) and (2008), Roemer (2011) invokes the authority of an "ethical observer" to reject any moral theory involving a positive rate of time preference. Dasgupta's (2011) response opposes the moral fundamentalism imposed by an external, overarching "ethical observer" in favor of approaches to intergenerational justice grounded in the here-and-now judgment of a "social evaluator" (i.e., "the concerned citizen or responsible public decision-maker").[17] We take a similar view here: in promoting intergenerational neutrality, the social evaluator can be seen as

[16] Deaton (2007) refers to these positions as the *English* and *American schools* respectively.

[17] Dasgupta (2011, 2008, 2005) rejects classical utilitarianism on the grounds that its ex cathedra pronouncements on intergenerational justice always trump other value judgments that may be valid under alternative ethical frameworks. Instead, he is more sympathetic to "intuitionism," which permits situation-dependent judgments, often involving competing ethical values (e.g., intergenerational neutrality vs. consumer sovereignty). We do not try to resolve these issues here. Rather, we explore the consequences for sustainable growth of Koopmans (1965) framework, which is not necessarily utilitarian, and his 'ethical preference for neutrality'.

adopting the view of the concerned citizen from behind the veil of ignorance in the original position, i.e. a position that forces participants in society to select principles impartially and rationally (Rawls 1971). Moreover, Dasgupta observes that intertemporal welfare economics generally fails to distinguish discounting as a determinant of an individual's lifetime welfare from that used for intergenerational welfare. Dietz and Asheim (2012) also observe that the climate change literature overlooks this distinction. A key objective of the current paper is to clarify the distinction between individual intertemporal choice and intergenerational justice, with implications for sustainability.[18]

3. General model and results

Burton (1993) examined implications of distinguishing intra- and inter-generational discounting for optimal resource extraction in a cake-eating economy. In what follows, we use a similar distinction to examine issues relating to sustainable growth albeit in a production economy with produced capital and renewable resources. This allows the planner to respect consumer sovereignty regarding impatience within a generation while simultaneously imposing different weighting between generations. Production is given by $Y = F(K, R)$, where K is the stock of capital that depreciates at a constant rate, γ, and R is the extraction from a renewable natural resource stock, X, that grows at rate $G(X)$, and is extracted at a stock-dependent unit cost, $\theta(X)$.

We assume that the economy is made up of overlapping generations of otherwise identical individuals. In the simplest representation, each generation contains one individual

[18] Dasgupta (2011) mentions work in progress by Dasgupta and Maskin allowing for a different, presumably lower, intergenerational discount rate.

who lives to age *N*. At each time *t*, society is made of individuals who range in age from *0* to *N*, and no two individuals have the same age. An individual of age τ is allotted consumption good in amount $c(t,\tau)$ and enjoys utility $u(c(t,\tau))$. We assume identical utility functions across individuals and time periods.

An individual born at time *T* measures remaining lifetime utility, U_T, according to the formula, where β is utility discount rate for individuals.

$$U_T = \int_{\tau=0}^{N} u(c(T+\tau,\tau))e^{-\beta\tau}\,d\tau, \qquad -N \le T. \tag{1}$$

Social welfare is then a weighted sum of the lifetime utilities of individual members of society, where the weights are based on the generational discount rate, ρ.

$$\widetilde{W} = \int_{T=-N}^{\infty} U_T e^{-\rho T}\,dT. \tag{2}$$

Thus, the social planner's problem is to choose consumption for each individual at each time $c(t,\tau), \forall t,\tau$ (a time path for each individual's consumption) subject to the equations of motion of the two stocks, i.e.,

$$\begin{aligned} &\underset{c(t,\tau)}{Max}\,\widetilde{W},\ \widetilde{W} = \int_{T=-N}^{\infty} U_T e^{-\rho T}\,dT \\ &s.t.\ \dot{K} = F(K,R) - \gamma K - \theta(X)R - \int_0^N c(t,\tau)d\tau,\ K(0) = K_0 \\ &\qquad \dot{X} = G(X) - R,\ X(0) = X_0 \end{aligned} \tag{3}$$

This can be solved as an optimal control problem. Since the two equations of motion are describing rates of change in terms of pure time, *t*, application of the Maximum Principle to this problem can be greatly simplified by reformulating the objective function in terms of time, *t*, instead of the generational index, *T*. For this purpose, we switch from separability of social welfare by individual to separability by time period through a transformation of the two timevariable system from (T,τ) to (t,τ) by setting $T = t - \tau$ and maintaining $\tau = \tau$. This is consistent with Burton's (1993) welfare trade-off analysis, involving both generational and personal discount factors. We can then re-write the optimization problem (3) as[19]:

$$\begin{aligned} &Max\, W,\; W = \int_0^\infty V(C) e^{-\rho t} dt \\ &s.t.\; \dot{K} = F(K,R) - \gamma K - \theta(X)R - C(t),\; K(0) = K_0 \\ &\quad\;\; \dot{X} = G(X) - R,\; X(0) = X_0 \end{aligned} \tag{4}$$

where

$$\begin{aligned} &V(C(t)) = Max \int_0^N u(c(t,\tau)) e^{-(\beta-\rho)\tau} d\tau \\ &s.t. \int_0^N c(t,\tau) d\tau \le C(t) \end{aligned}$$

$C(t)$ represents the aggregate level of output available for consumption at time *t*, and *V* represents aggregate utility.

This can be solved as a 2-stage maximization problem. The first stage establishes a relationship between $c^*(t,0)$ and $c^*(t,\tau)$, the optimal distribution of individual consumption across generation at time t. The first stage also establishes a relationship between $c^*(t,0)$ and aggregate consumption $C(t)$. The second stage determines the path of optimal aggregate consumption $C^*(t)$, the solution to the planner's problem (4), which involves η and ρ, but not β.

First, we provide the first order conditions from the maximization problem that determines *V*. To solve for $V(C)$, form the Lagrangian:

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[19] Similar to Burton's formulation (Burton (1993), note 6, p123), *W* differs from $\tilde{W}$ by a constant *k*, that portion of lifetime utility of existing cohorts that was received in past periods.

$$L = \int_0^N u\left(c\left(t,\tau\right)\right)e^{-(\beta-\rho)\tau}\,d\tau + \lambda\left[C(t) - \int_0^N c(t,\tau)d\tau\right] \tag{5}$$

Inspection of the Lagrangian indicates that a necessary condition for maximization is that the weighted marginal utilities be equated, so that along the optimum consumption path, c^* satisfies

$$u'\left(c^*\left(t,\tau\right)\right) = u'\left(c^*\left(t,0\right)\right)e^{(\beta-\rho)\tau}, \qquad 0 \le \tau \le N \tag{6}$$

From equation (6), each generation's share of aggregate consumption is determined by the difference between the intra- and inter-generational discount rates. Suppose $\beta > \rho$, i.e., the personal discount rate is greater than the generational discount rate. It then follows directly from equation (6) that $u'(c^*(t,\tau)) > u'(c^*(t,0))$, i.e., that marginal utility is lower for younger individuals and $c^*(t,\tau) < c^*(t,0)$. That is, at any time t, a younger individual consumes a larger share of society's aggregate consumption than an older one. On the other hand, if the intergenerational discount rate is higher than the intragenerational one (i.e., $\beta < \rho$), a younger individual will consume a smaller share of aggregate consumption. All generations enjoy the same share when intra- and inter-generational discount rates are equal ($\beta = \rho$).[20]

Following the standard approach in neoclassical sustainable growth theory, we now assume that the felicity function takes the constant elasticity of marginal felicity form,[21]

$$u(c(t,\tau)) = -[c(t,\tau)]^{-(\eta-1)}, \qquad \eta > 1. \tag{7}$$

[20] Burton (1993) and Schneider et al. (2012) make similar observations.

[21] See Arrow (1965 cited in Dasgupta 2011) and Dasgupta (2011) on the merits of studying the increasing elasticity case. This CES form follows Dasgupta and Heal (1979). Using a more general functional form yields the identical form of the Ramsey equation, but makes the derivation more tedious.

Equation (7) then implies that at the optimum,

$$c^*(t,\tau) = c^*(t,0)e^{-(\beta-\rho)\tau/\eta} \tag{8}$$

As shown in Appendix I, this functional specification permits the computation of the (planner's) utility of aggregate consumption:

$$V(C) = -AC^{-(\eta-1)}, \tag{9}$$

where A represents the "aggregation coefficient", which depends on the parameters β, ρ, and η.

We now form the current-value Hamiltonian for maximizing W:

$$H = V(C) + \lambda[F(K,R) - \gamma K - \theta(X)R - C] + \psi[G(X) - R]. \tag{10}$$

Application of the maximum principle yields the standard first order conditions, which can be manipulated to generate the Ramsey condition and the generalized form of the Hotelling rule for renewable resources:[22]

$$\frac{V''(C)}{V'(C)}\dot{C} = \rho - (F_k - \gamma) \tag{11}$$

and

$$[F_R - \theta(X)] = \frac{1}{F_k - \gamma - \rho}\left\{\dot{F}_R + [F_R - \theta(X)]G'(X) - \theta'(X)G(X)\right\} \tag{12}$$

Further development of equation (11) is especially revealing. Computing derivatives and then forming the ratio V''/V' results in cancellation of the coefficient A.[23] The Ramsey condition can then be written in the standard way

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[22] For a non-renewable resource, equation (12) reduces to: $[F_R - \theta(X)] = (F_k - \gamma - \rho)^{-1}\dot{F}_R$.

[23] As a coefficient, A represents a monotonic transformation of utility of aggregate consumption and could have been divided out in the original specification of $V(C)$.

$$F_k - \gamma = \eta \frac{\dot{C}}{C} + \rho \qquad (13)$$

Arbitrage conditions (12) and (13) show that even in this overlapping generations model, the optimum trajectory of aggregate consumption is governed at each time by the relationships among aggregate quantities and the generational discount rate, ρ, but not the personal discount rate, β.[24]

In steady-state:

$$\begin{aligned} &\dot{C} = 0 \Rightarrow \eta \frac{\dot{C}}{C} = F_k - (\gamma + \rho) = 0 \\ &\Rightarrow F_k = (\gamma + \rho) \end{aligned} \qquad (14)$$

Thus, if the optimal trajectory involves capital accumulation over time, then before reaching the steady-state, we have $F_k > (\gamma + \rho)$ and, therefore, $\dot{C} > 0$.

Solution of the planner's problem yields optimum aggregate consumption $C^*(t)$ at each time t. This result permits return to stage 1 optimization to determine the distribution of individual consumption across generations.

From Appendix I, we have $C^*(t) = Mc^*(t,0)$, or

$$\begin{aligned} c^*(t,0) &= \frac{1}{M} C^*(t) \\ &= \overline{M} C^*(t), \quad \textit{where} \;\; \overline{M} = \frac{1}{M} = \frac{(\beta - \rho)}{\eta(1 - e^{-(\beta-\rho)N/\eta})} \end{aligned} \qquad (15)$$

Comparative static analysis yields

[24] Calvo and Obstfeld (1988), Marini and Scaramozzino (1995) and Schneider et al. (2012) also show that for the case of homogeneous agents, the continuous time OLG framework collapses to the standard representative agent model.

$$\frac{\partial c^*(t,0)}{\partial \beta} = C^*(t)\frac{\partial \overline{M}}{\partial \beta} > 0, \tag{16}$$

as established in Appendix I. The result that the planner's solution is independent of β does not suggest that individual impatience fails to factor into consumption decisions. On the contrary, raising β increases $c^*(t,0)$, tilting individual consumption towards younger generations, compensated for, in the aggregate, by decreasing consumption of older generations. In other words for a given ρ, the PV-maximizing aggregate consumption trajectory can always be achieved by adjusting individual consumption profiles in response to perturbations of β.

4. The case of intergenerational neutrality

We believe that this finding has important implications for modeling economic growth in a manner compatible with intergenerational equity. Individual impatience does not conflict with the planner's desire to establish normative weighting of generation. In particular, as suggested by the quotes in the introduction, stewardship for the future can be accommodated by setting the generational discount rate, ρ, equal to zero. The immediate problem with this approach is that the welfare maximand is infinite for any consumption path that does not converge to zero. As shown in Endress et al. (2005), this objection can be readily overcome, however, by transforming the objective function *ala* Koopmans (1965) as:

$$\underset{C(t)}{Max} \; W, \quad W = \int_{t=0}^{\infty} \left[V(C(t)) - V(\hat{C}) \right] dt \tag{17}$$

where $\hat{C}$ is golden rule consumption.

Now, the Hamiltonian becomes:

$$H = [V(C(t)) - V(\hat{C})] + \lambda[F(K,R) - \gamma K - \theta(X)R - C] + \psi[G(X) - R] \quad (18)$$

The Hamiltonian (18) is an autonomous control problem. Following Chiang (1992) (equations 8.15 – 8.17), we have, $\frac{\partial H}{\partial t} = 0$; along the optimal path, $\frac{dH}{dt} = \frac{\partial H}{\partial t}$ so that $H = 0, \forall t$. Intuitively, each of the expressions in the square brackets approaches zero in the steady state, which implies that *H*=0 in the steady state and in every other period. Rearranging (18) yields

$$\begin{aligned} V(\hat{C}) &= V(C(t)) + \lambda[F(K,R) - \gamma K - \theta R - C] + \psi[G(X) - R] \\ \Rightarrow V(\hat{C}) &= V(C(t)) + \lambda \dot{K} + \psi \dot{X} \end{aligned} \quad (19)$$

That is, sustainable income defined as the maximum value of social utility at any point in time is equal to the value of consumption and net investment, evaluated at the shadow prices of produced and natural capital. The latter provides a definition of GNNP (see e.g. Weitzman, 1976, 1997). Under intergenerational neutrality while simultaneously allowing for individual impatience, GNNP is sustained forever. This avoids the conundrum in models with positive intergenerational discounting that optimal consumption and sustainable income are eventually declining.

5. Extension: renewable resource with amenity value

Gerlagh and Keyser (2001) present a discrete-time OLG model with a single exhaustible resource that has amenity value to compare policies for intergenerational distribution of natural resource entitlements. The motivation for including amenity value of the resource, based on the discussion in Krautkraemer (1985), is the observation that resource extraction often results in environmental degradation (e.g. strip mining, fracking). There is no backstop technology in the Gerlagh and Keyer (2001) model. Instead, the resource is modeled as non-essential in production, which allows the model to arrive at a steady state level of amenity. The authors cite Bovenberg and Heijdra (1998), which is closer to our paper with a renewable resource and follows Blanchard (1985), but not Calvo and Obstfeld (1988). Bovenberg and Heijdra aggregate variables, but don't consider the distinction between personal and generational rates of time preference. The key result is an intergenerational transfer mechanism that Gerlagh and Keyser (2001) characterize as "a centrally planned and relatively sophisticated dynamic adjustment of public debt." Our approach to intergenerational neutrality is more general, and at the same time, less complex.

When the resource stock is modeled to include amenity value, amenity is non-rival, and utility is separable in *C* and *X*, we find that the standard Ramsey condition (13) can be derived from the first order conditions of the modified optimization problem. The Hotelling rule, however, must be modified to include the marginal rate of substitution between *X* and *C*:[25]

$$[F_R - \theta(X)] = \frac{1}{F_K - \gamma - \rho}\left\{\dot{F}_R + [F_R - \theta(X)][G'(X) - \rho] - \theta'(X)G(X) + \frac{U_X}{U_C}\right\} \tag{20}$$

In the steady state, $\dot{K} = 0$, $\dot{C} = 0$ and $\dot{F}_R = 0$, which means that the modified golden rule conditions are

$$F_K = (\gamma + \rho) \tag{21}$$

and

$$F_R = \theta(X) + \frac{\theta'(X)G(X) - U_X / U_C}{G'(X) - \rho} \tag{22}$$

The golden rule conditions are analogous, except with $\rho = 0$, and can be solved for $\hat{K}$ and $\hat{R}$, which in turn determine $\hat{C}$ and $\hat{X}$. The Koopmans transformation is then $U(C, X) - U(\hat{C}, \hat{X})$.

[25] For detailed derivations of these and any other results discussed in this section, see appendix II.

The Ramsey and Hotelling conditions in transition to the steady state are as before, but with $\rho = 0$. Using a first order approximation, it is straightforward to show that the *MRS* term can be reduced to $[N^{-(\eta-1)}][(C/X)^{-\eta}]$. Even with the inclusion of amenity, the continuous time OLG framework "collapses" to the standard Ramsey-Hotelling model and individual impatience drops out.

Now suppose a tax *T* is placed on *X* to help restore resource quality, which may be degraded as amenity services are provided (e.g., eco-tourism). The tax revenue $T \cdot X$ goes entirely toward restoration, and the coefficient *α*, $0 \le \alpha \le 1$, accounts for possible partial restoration. The new current value Hamiltonian is

$$H = U(C,X) + \lambda[F(K,R) - \gamma K - \theta(X)R - T \cdot X - C] + \psi[G(X) + \alpha T \cdot X - R] \qquad (23)$$

It is straightforward to derive the standard Ramsey condition, but the generalized form of the Hotelling rule turns out to be more complex:

$$F_R - \theta(X) = \frac{\dot{F}_R - \theta'(X)[G(X) + \alpha T] + U_X / U_C - T + [F_R - \theta(X)][G'(X) + \alpha T - \rho]}{F_K - \gamma - \rho} \qquad (24)$$

After some algebraic manipulation, one can show that the amenity tax which approximates intergenerational neutrality is

$$T = \rho \cdot D,\ D = \frac{[\theta'(X)G(X) - U_X / U_C]}{\alpha[\theta'(X)G(X) - \theta'(X)G'(X)X - U_X / U_C] - G'(X)} \qquad (25)$$

To explore the effects of varying production and cost parameters on the tax coefficient *D*, we work through a simple numerical example. In order to obtain specific solutions, we make the following assumptions: $G(X) = rX(1 - X / X_{\max})$; $F(K,R) = K^a R^b,\ a + b = 1$; $U(C,X) = \log C + \log X$; $\theta(X) = e / X$; *γ* constant, and *α*=1. Without loss of generality, we can also set $r = 1$ and $X_{\max} = 1$ because *r* drops out in the computation of steady state $\hat{X}$ and MRS, and $\hat{X}$ is computed as a function of $X_{\max}$. We compute the amenity tax coefficient *D* for various values of *e* and *a* while holding capital depreciation fixed at $\gamma = 1/10$ (Table 1). We also compute $\widetilde{D}$ for the assumption that $\theta'(X) = 0$.

Table 1. Amenity tax sensitivity analysis

	e=1/10			e=1/2			e=1		
	a=1/3	a=1/2	a=2/3	a=1/3	a=1/2	a=2/3	a=1/3	a=1/2	a=2/3
D	4.86	1.55	1.06	2.56	1.23	1.02	0.570	0.785	0.966
$\widetilde{D}(\theta' = 0)$	15.50	1.74	1.07	-0.887	1.66	1.07	0.218	∞	1.08

For what we believe is a reasonable representation of production (*a*=2/3), the results are not heavily sensitive to *e*. $\hat{X}$ increases slightly with *e* but remains close to 2/3. That means MRS dominates in the formula for *D* and $D \approx 1$ for all values of *e* tested. Since $\widetilde{D} \approx 1$ for all values of *e* as well, we conclude that for $a > b$, $\widetilde{D}$ is a reasonable approximation. For this particular case, the approximate amenity tax is $T = \rho$ (if there is a positive ρ). For the case of *a*=2/3 and $\alpha < 1$, where the magnitude of MRS dominates the magnitude of $G'(X)$, the approximate amenity tax becomes $T = \rho / \alpha$. The smaller the restoration coefficient, the higher is the amenity tax.

6. Conclusions

We develop a model of optimal and sustainable growth that accommodates distinct inter- and intra-generational discount rates. Under the assumption of constant elasticity of marginal felicity, the optimum trajectory of aggregate consumption is guided, via the Ramsey condition, by the intergenerational discount rate but not the personal discount rate. As the personal discount rate increases, each generation allocates a higher share of its lifetime consumption to earlier stages of life. However, an increase in the personal discount rate does not affect the trajectory of aggregate consumption.

We also show that the optimal consumption trajectory is continually rising, provided that society is approaching golden rule consumption from a stock of physical capital lower than the steady state quantity. This implies that a sustainability constraint, if imposed, would be non-binding. We also show that sustainable income, defined as GNNP, is actually sustained along the optimal path, avoiding an apparent paradox in the case of positive generational discounting.

When the resource stock is modeled to include amenity value, amenity is non-rival, and utility is separable in consumption and amenity, we find that a standard Ramsey condition can be derived, and a modified Hotelling condition, which depends on the MRS, governs optimal resource extraction. Results from a numerical exercise suggest that an amenity tax designed to approximate intergeneration neutrality is not very sensitive to resource extraction costs. Moreover, when the MRS dominates resource growth effects, the amenity tax takes a particularly simple form – the product of the planner's rate of time preference and the inverse of the restoration coefficient.

For project evaluation of large investment projects such as climate-change mitigation, it may be informative to ask to what extent intragenerational impatience would change such findings as in *Stern Review* (2006), even in the presence of intergenerational neutrality.
Just as we have distinguished between the intra- and inter-generational utility discount rates, one could further distinguish between the planner's intergenerational inequality aversion and the intragenerational elasticity, η, as revealed by market transactions. We leave this exploration to further research.

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Appendix I

Utility of aggregate consumption in a model of overlapping generations

Assume that individual utility of consumption has a functional form with constant elasticity of marginal utility:

$$u(c(t,\tau)) = -[c(t,\tau)]^{-(\eta-1)}, \eta > 1$$

Then $u'[c(t,\tau)] = (\eta-1)[c(t,\tau)]^{-\eta}$

Efficiency condition (6) can then be written as

$$(\eta-1)[c^*(t,\tau)]^{-\eta} = (\eta-1)[c^*(t,0)]^{-\eta} e^{(\beta-\rho)\tau}$$

or $c^*(t,\tau) = c^*(t,0)e^{-(\beta-\rho)\tau/\eta}$

Substitute in the consumption constraint of (5) to solve for the aggregate consumption, $C(t)$:

$$C(t) = \int_0^N c^*(t,\tau)d\tau$$

$$= \int_0^N c^*(t,0)e^{-(\beta-\rho)\tau/\eta}d\tau$$

$$= \left\{\frac{\eta(1-e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)}\right\} c^*(t,0)$$

$$= Mc^*(t,0),$$

where the constant $M = \left\{\frac{\eta(1-e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)}\right\}$

Now $V(C(t)) = Max\int_0^N u(c(t,\tau))e^{-(\beta-\rho)\tau}d\tau$

$$= \int_0^N u(c^*(t,\tau))e^{-(\beta-\rho)\tau}d\tau$$

But, $u(c^*(t,\tau)) = -[c^*(t,\tau)]^{-(\eta-1)}$

$$= -[c^*(t,0)e^{-(\beta-\rho)\tau/\eta}]^{-(\eta-1)}$$

$$= -[c^*(t,0)]^{-(\eta-1)}e^{(\beta-\rho)(\eta-1)\tau/\eta}$$

$$= -\left\{\frac{C(t)}{M}\right\}^{-(\eta-1)} e^{(\beta-\rho)(\eta-1)\tau/\eta}$$

So, $V(C(t)) = -\left\{\frac{C(t)}{M}\right\}^{-(\eta-1)} \int_0^N e^{(\beta-\rho)(\eta-1)\tau/\eta} e^{-(\beta-\rho)\tau} d\tau$

$$= -\left\{\frac{C(t)}{M}\right\}^{-(\eta-1)} \int_0^N e^{-(\beta-\rho)\tau/\eta} d\tau$$

$$= -\left\{\frac{C(t)}{M}\right\}^{-(\eta-1)} \left[\frac{\eta(1-e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)}\right]$$

$$= -\left\{\frac{C(t)}{M}\right\}^{-(\eta-1)} \cdot M$$

$$= -M^{\eta}[C(t)]^{-(\eta-1)}$$

$$= -A[C(t)]^{-(\eta-1)}$$

where, $A = M^{\eta}$

Comparative statics

We wish to show that $\frac{\partial \overline{M}}{\partial \beta} > 0$, where $\overline{M} = \frac{1}{M}$. We examine the expression

$\overline{M} = \frac{x}{\eta(1-e^{-bx})}$, where $x = (\beta-\rho)$ and $b = \frac{N}{\eta}$.

Now, $\frac{\partial \overline{M}}{\partial x} = \frac{1-(1+bx)e^{-bx}}{\eta(1-e^{-bx})^2}$.

Thus, it is sufficient to determine the sign of the numerator. It is easy to show that the expression $(1+bx)e^{-bx}$ attains a maximum of 1 for $x = 0$ and is monotonically decreasing thereafter. Thus for $x \neq 0$, the numerator is positive so that $\frac{\partial \overline{M}}{\partial x} > 0$.

Appendix II

Amenity value of the resource stock is treated as non-rival and the utility function is separable:

$$U(C,X) = V(C) + \Omega(X) \tag{A1}$$

The maximization problem for the modified problem is

$$\begin{aligned}
&\text{Max}\, W,\ W = \int_0^\infty U(C,X)e^{-\rho t}dt \\
&\text{s.t. } \dot{K} = F(K,R) - \gamma K - \theta(X)R - C,\ K(0) = K_0 \\
&\qquad \dot{X} = G(X) - R,\ X(0) = X_0,\ 0 \le X \le X_{\max}
\end{aligned} \tag{A2}$$

where

$$\begin{aligned}
&U(C,X) = \{V(C) + \Omega(X)\} = \text{Max}\int_0^N \{u(c(t,\tau)) + \omega(X(t,\tau))\}e^{-(\beta-\rho)\tau}d\tau \\
&\text{s.t. } \int_0^N c(t,\tau)d\tau \le C(t),\ X(t,\tau) = X(t) \text{ for all } \tau
\end{aligned} \tag{A3}$$

The separability assumption permits one to solve the two components of the integrand separately. The first piece has already been solved in the standard framework, and the second piece is straightforward to solve, given that amenity is treated as non-rival:

$$\begin{aligned}
\Omega(X) &= \text{Max}\int_0^N \omega(X(t,\tau))e^{-(\beta-\rho)\tau}d\tau = \int_0^N \omega(X(t))e^{-(\beta-\rho)\tau}d\tau \\
&= \omega(X(t))\int_0^N e^{-(\beta-\rho)\tau}d\tau = \omega(X(t))\frac{1}{\beta-\rho}\left[1-e^{-(\beta-\rho)N}\right] \\
&= B\cdot\omega(X(t))
\end{aligned} \tag{A4}$$

where $B = (\beta-\rho)^{-1}[1-e^{-(\beta-\rho)N}]$. If we suppose that $\omega(X(t)) = -[X(t)]^{-(\eta-1)}$, then

$$\Omega(X) = -B\cdot[X(t)]^{-(\eta-1)} \tag{A5}$$

We also already know that

$$V(C) = -A\cdot[C(t)]^{-(\eta-1)} \tag{A6}$$

where $A=\{(\beta-\rho)^{-1}\eta[1-e^{-(\beta-\rho)N/\eta}]\}^{\eta}$. In general, $A\neq B$ because of how rival and non-rival elements are treated across groups τ at time t. Equation (A3) can now be rewritten:

$$U(C,X)=-A[C(t)]^{-(\eta-1)}-B[X(t)]^{-(\eta-1)} \tag{A7}$$

The current value Hamiltonian corresponding to the dynamic optimization problem (A2) is

$$H=U(C,X)+\lambda[F(K,R)-\gamma K-\theta(X)R-C]+\psi[G(X)-R] \tag{A8}$$

Application of the maximum principle yields standard first order conditions, which can be used to derive the Ramsey condition:

$$(F_K-\gamma)=\rho-\frac{V''(C)}{V'(C)}\dot{C}=\rho+\eta\frac{\dot{C}}{C} \tag{A9}$$

and a Hotelling rule for extraction of the renewable resource:

$$[F_R-\theta(X)]=\frac{1}{F_K-\gamma-\rho}\left\{\dot{F}_R+[F_R-\theta(X)][G'(X)-\rho]-\theta'(X)G(X)+\frac{U_X}{U_C}\right\} \tag{A10}$$

where the marginal rate of substitution is

$$MRS=\frac{U_X}{U_C}=\frac{B}{A}\left(\frac{X}{C}\right)^{-\eta} \tag{A11}$$

Although β, which is embedded in A, disappears from the Ramsey equation, the constant $(B/A)\neq 1$ in general and embodies β, ρ and η.

In the steady state, $\dot{K}=0$, $\dot{C}=0$ and $\dot{F}_R=0$, which means that the modified golden rule conditions are

$$F_K=(\gamma+\rho) \tag{A12}$$

and

$$F_R=\theta(X)+\frac{\theta'(X)G(X)-U_X/U_C}{G'(X)-\rho} \tag{A13}$$

The golden rule conditions are analogous, except with $\rho = 0$, and can be solved for $\hat{K}$ and $\hat{R}$, which in turn determine $\hat{C}$ and $\hat{X}$. The Koopmans transformation is then $U(C,X) - U(\hat{C},\hat{X})$. The Ramsey and Hotelling conditions in transition to the steady state are as before, but with $\rho = 0$.

Recall that $A = \{(\beta-\rho)^{-1}\eta[1-e^{-(\beta-\rho)N/\eta}]\}^{\eta}$ and $B = (\beta-\rho)^{-1}[1-e^{-(\beta-\rho)N}]$. If we use the first order approximation $e^{-z} = (1-z)$, *A* and *B* simplify to $A = N^{\eta}$ and $B = N$, and the expression for *MRS* in the Hotelling condition becomes $[N^{-(\eta-1)}][(C/X)^{-\eta}]$, where the first term in the square brackets is equal to A/B. Even with the inclusion of amenity, the continuous time OLG framework "collapses" to the standard Ramsey-Hotelling model and individual impatience drops out.

Now suppose a tax *T* is placed on amenity *X* to help restore resource quality. The tax revenue $T \cdot X$ goes entirely toward restoration, and the coefficient *α*, $0 \le \alpha \le 1$, accounts for possible partial restoration. The new planner's problem is

$$\begin{aligned} &\text{Max}\int_0^{\infty} U(C,X)e^{-\rho t}dt \\ &\text{s.t. } \dot{K} = F(K,R) - \gamma K - \theta(X)R - T\cdot X - C,\ K(0) = K_0 \\ &\qquad \dot{X} = G(X) + \alpha T\cdot X - R,\ X(0) = X_0,\ 0 \le X \le X_{\max} \end{aligned} \tag{A14}$$

The corresponding current value Hamiltonian is

$$H = U(C,X) + \lambda[F(K,R) - \gamma K - \theta(X)R - T\cdot X - C] + \psi[G(X) + \alpha T\cdot X - R] \tag{A15}$$

It is straightforward to derive the standard Ramsey condition, but the generalized form of the Hotelling rule turns out to be more complex. Taking the time derivative of the expression for *ψ* in the first order condition for *R* yields

$$\dot{\psi} = U_C[\dot{F}_R - \theta'(X)G(X) - \theta'(X)\alpha T \cdot X + \theta'(X)R] - U_C[F_R - \theta(X)][F_K - \gamma - \rho] \tag{A16}$$

Substituting (A16) and the expression for ψ into the first order condition for X results in the following expression:

$$\begin{aligned}&[\dot{F}_R - \theta'(X)G(X) - \theta'(X)\alpha T] - [F_R - \theta(X)][F_K - \gamma - \rho] \\ &= -U_X / U_C + T - [F_R - \theta(X)][G'(X) + \alpha T - \rho]\end{aligned} \tag{A17}$$

which can be rearranged into the generalized form of the Hotelling rule for renewable resources:

$$F_R - \theta(X) = \frac{\dot{F}_R - \theta'(X)[G(X) + \alpha T] + U_X / U_C - T + [F_R - \theta(X)][G'(X) + \alpha T - \rho]}{F_K - \gamma - \rho} \tag{A18}$$

In the steady state, $\dot{F}_R = 0$ and $F_K = \gamma + \rho$, which means that

$$F_R - \theta(X) = \frac{\theta'(X)[G(X) + \alpha T] + T - U_X / U_C}{G'(X) + \alpha T - \rho} \qquad \text{A19)}$$

The objective of the amenity tax is to approximate intergenerational neutrality, i.e. the tax should induce the golden rule steady state that obtains for $\rho = 0$. Thus, we solve for tax T that makes the steady state with $\rho > 0$ equal to the golden rule steady state:

$$\frac{\theta'(X)[G(X) + \alpha T \cdot X] + T - U_X / U_C}{G'(X) + \alpha T - \rho} = \frac{\theta'(X)G(X) - U_X / U_C}{G'(X)} \tag{A20}$$

After some algebraic manipulation the optimal amenity tax is

$$T = \rho \cdot D, \ D = \frac{[\theta'(X)G(X) - U_X / U_C]}{\alpha[\theta'(X)G(X) - \theta'(X)G'(X)X - U_X / U_C] - G'(X)} \tag{A21}$$

When harvesting cost is constant, i.e. $\theta'(X) = 0$, the tax becomes

$$T = \rho \cdot \frac{U_X / U_C}{\alpha U_X / U_C + G'(X)} \tag{A22}$$

To explore the effects of varying production and cost parameters on the tax coefficient D, we work through a simple numerical example. In order to obtain specific solutions, we make the following assumptions: $G(X) = rX(1 - X / X_{\max})$; $F(K,R) = K^a R^b$, $a + b = 1$;

$U(C,X) = \log C + \log X$; $\theta(X) = e / X$; γ constant, and α=1. Without loss of generality, we can set $r = 1$ and $X_{\max} = 1$ because r drops out in the computation of steady state $\hat{X}$ and MRS, and $\hat{X}$ is computed as a function of $X_{\max}$. Then $F_K = aK^{a-1}R^b$, $F_R = bK^a R^{b-1}$, $G'(X) = 1 - 2X$, $\theta'(X) = -eX^{-2}$ and $U_X / U_C = C / X$. In the Golden Rule steady state, $F_K = \gamma$, which implies that $R = K(\gamma / a)^{1/b}$ and $F_R = b(\gamma / a)^{-a/b}$. We use $\overline{F}_R$ to designate the golden rule steady state value of F_R. Also since $\dot{K} = 0$, $C = F(K,r) - \gamma K - \theta(X)R$, or equivalently

$$C = R\left(\frac{\gamma}{a}\right)^{-\frac{a}{b}} - Ra\left(\frac{\gamma}{a}\right)^{-\frac{a}{b}} - R\left(\frac{e}{X}\right) \tag{A23}$$

In the steady state $\dot{X} = 0$, which implies that $R = G(X) = X(1 - X)$. Plugging R into (A23) yields

$$C = \overline{F}_R X(1 - X) - e(1 - X) \tag{A24}$$

The Golden Rule steady state (with $\rho = 0$) is described by

$$\overline{F}_R - \theta(\hat{X}) = \frac{\theta'(\hat{X})G(\hat{X}) - MRS}{G'(\hat{X})} \Rightarrow \hat{X} = \frac{(\overline{F}_R + e) \pm \sqrt{(\overline{F}_R)^2 + e^2 - e\overline{F}_R}}{3\overline{F}_R} \tag{A25}$$

We compute the amenity tax coefficient D for various values of e and a while holding capital depreciation fixed at $\gamma = 1/10$ (Table A1). We also compute $\widetilde{D}$ for the assumption that $\theta'(X) = 0$:

$$\widetilde{D} = \frac{2\overline{F}_R - 3e}{2[\overline{F}_R - 1] - 3e} \tag{A26}$$

For what we believe is a reasonable representation of production (a=2/3), the results are not heavily sensitive to e. $\hat{X}$ increases slightly with e but remains close to 2/3. That means MRS dominates in the formula for D and $D \approx 1$ for all values of e tested. Since $\widetilde{D} \approx 1$ for all values of e as well, we conclude that for $a > b$, $\widetilde{D}$ is a reasonable approximation. For this particular case, the approximate amenity tax is $T = \rho$ (if there is a positive ρ). For the case of a=2/3 and $\alpha < 1$, where the magnitude of MRS dominates the magnitude of $G'(X)$, the tax formula (A22) collapses to $T = \rho / \alpha$. The smaller the restoration coefficient, the higher is the amenity tax.

Table A1. Amenity tax sensitivity analysis

	e=1/10			e=1/2			e=1		
	a=1/3	a=1/2	a=2/3	a=1/3	a=1/2	a=2/3	a=1/3	a=1/2	a=2/3
$\overline{F}_R$	1.22	2.50	14.81	1.22	2.50	14.81	1.22	2.50	14.81
$\hat{X}$	0.681	0.673	0.668	0.760	0.706	0.672	0.914	0.757	0.679
$\theta(\hat{X})$	0.147	0.149	0.150	0.658	0.708	0.744	1.09	1.32	1.47
$\theta'(\hat{X})$	-0.216	-0.221	-0.224	-0.866	-1.00	-1.11	-1.20	-1.75	-2.17
$G(\hat{X})$	0.217	0.220	0.222	0.182	0.208	0.220	0.079	0.184	0.218
$G'(\hat{X})$	-0.362	-0.346	-0.336	-0.520	-0.412	-0.344	-0.828	-0.514	-0.358
$\theta'(\hat{X})G(\hat{X})$	-0.047	-0.049	-0.050	-0.158	-0.208	-0.244	-0.095	-0.322	-0.473
$\theta'(\hat{X})G'(\hat{X})\hat{X}$	0.053	0.051	0.050	0.342	0.291	0.257	0.908	0.681	0.527
$MRS = C / \hat{X}$	0.342	0.769	4.87	0.134	0.439	4.61	0.011	0.287	4.28
D	4.86	1.55	1.06	2.56	1.23	1.02	0.570	0.785	0.966
$\widetilde{D}(\theta' = 0)$	15.50	1.74	1.07	-0.887	1.66	1.07	0.218	∞	1.08