Uncertain Populations and the Value of Information

1. Introduction

An emerging problem in natural resource policy is how to design efficient strategies for

managing invasive species. Invasive species, those plants, animals, and microbes that are

nonnative to an area and have caused or have the potential to cause economic and or

ecological damage threaten natural resources, biodiversity, and human health worldwide.[1]

Damages from invasive species are ecological as well as economic. These include lost

biodiversity and reduced ecosystem services, as well as direct and indirect economic

damages such as health damages or lost productivity. Caterpillars from the Asian gypsy

moth (*Lymantria dispar*) cause extensive defoliation, reduced growth and mortality of

host trees throughout the northern hemisphere, while hairs on larvae and egg masses lead

to allergies in some people. The Nile perch *(Lates niloticus)* was introduced to Africa's

Lake Victoria in 1954 and has since contributed to the extinction of more than 200

endemic fish species through predation and competition for food. *Caulerpa taxifolia* is a

marine alga widely used as a decorative aquarium plant. The alga was accidentally

introduced into the Mediterranean Sea in wastewater, where it has now spread over more

than 13,000 hectares of seabed. This invader forms dense monocultures that prevent the

establishment of native seaweeds and exclude almost all marine life. Tamarisk (*Tamarix*

ramosissima) is a shrubby tree that can be found where its roots reach the water table,

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[1] This definition of invasive species is from President Clinton's Executive Order 13112, signed on February 3, 1999.

such as floodplains, along irrigation ditches and on lake shores. Tamarisk can tolerate a wide range of saline or alkaline soils and is able to dominate floodplain communities in the deserts of the Southwest United States due to its ability to tolerate water stress for extended periods of time. Tamarisk supports few native insects and thus is poor habitat for birds. The well-known invasion of the Brown treesnake (*Boiga irregularis*) on the island of Guam poses a real and immediate threat to the state of Hawaii, due to the large and increasing volume of military transport between to the locales, as well as commercial air and sea traffic. The snake has extirpated 11 native bird species on Guam, causes hundreds of hours of power outages a year, and sends a stream of citizens to the hospitals each year to treat venomous snakebites. Eight individual Brown treesnakes (hereafter, BTS) have been intercepted at the ports in Hawaii, accompanied by hundreds of credible snake sightings resulting in zero captures.

While the economic literature on invasive species has been growing rapidly, most analyses have made simplifying assumptions that may detract from the usefulness of their implications. One aspect of invasive species control that makes practical implementation particularly difficult is that the actual population of the species is almost never known. The only variables that a typical resource manager observes with certainty are the number of the invasive successfully harvested and the effort required to achieve that harvest. Like most renewable resource problems, the literature to date typically assumes a given initial population of the stock of interest. In our paper we develop a model in which the invasive species population is known neither in the initial period nor in any subsequent

management periods. Instead of setting harvest directly, managers set effort decisions for each period and then observe the harvest said effort yields.

In the context of renewable resource use including fisheries, several studies have analyzed the case where the resource stock is uncertain due to lack of information or measurement error (Clark and Kirkwood 1986, Roughgarden and Smith 1996, Sethi et al. 2005). Economic studies on biological invasion have focused on the case of deterministic species population. Knowler (2005) briefly mentions the possibility of uncertainty in resource management, but shows how it can be ignored when after-harvest populations can be set precisely as in Reed (1979). The only notable exceptions that we are aware of in the invasive species literature are Olson and Roy (2005) and Saphores and Shogren (2005). Invasion size is random in Olson and Roy, although after invasion the stock is perfectly observable. Furthermore, some of the simplifying assumptions required by their framework may misrepresent the challenges faced by real world planners in invasive species management, such as the linear growth function and the possibility of perfectly effective prevention. Saphores and Shogren allow growth to be uncertain, although stock is always accurately observed. Interestingly, they also allow for uncertainty regarding policy, where the optimal policy is not able to be computed until uncertain time T. While managers are (potentially inefficiently) investing funds to determine what the optimum policy is, the invasive population itself is left completely unmanaged. We depart from these existing models on invasive species management by assuming uncertain species population size.

Our model addresses two issues left unresolved in the literature to date. First, what is the best a manager can do when true optimum policy is unknown, and second, how can actual control efforts be used in place of costly research to inform policy making decisions.

Through species management, the managers obtain new information each period about the probability distribution of the species population size. Borrowing from the vast literature on renewable resources and learning, we model the connections between observable data (effort and harvest) and the unobservable invasive stock using the Bayesian methods. If the effort-harvest function is stochastic but known, the model allows for beliefs about the invasive population to be updated each period and the manager is therefore able to tailor the control strategy appropriately.

Section 2 sets up the baseline case where the population is known with certainty. Section 3 outlines the optimal strategy when the population is uncertain and considers a simplified functional form. Section 4 uses a case study, control of the Brown treesnake (*Boiga irregularis*) on the island of Saipan to illustrate the solution techniques. Section 5 concludes.

2. Harvest under Certainty

The usual renewable resource problem begins with a resource manager who maximizes the present value of a resource. Optimal management of an invasive species can be

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approached in a similar fashion. Maximizing the value of invasive species management is the same as minimizing the total present value of the expected costs of removing the species as well as the expected damages caused by the species.

In our model, the invasive species reproduces, causes damages, and is harvested in discrete time periods. We assume that population growth and ecological damages are deterministic, while the manager's harvest of the species is stochastic. The assumption of deterministic growth will help us isolate the information effects we are looking for later in the paper.

Let X_t denote the population of the invasive at time t. Each period this population causes $d(X_t)$ dollars worth of damage to the local ecosystem. Managers are able to reduce the population through stochastic harvesting. We denote e_t as the effort exerted to reduce the population, and $c(e_t)$ the cost the manager bears given e_t.

In the case where population is observable each period, the manager is able to set effort decisions based upon the observed harvest. The total present value (or cost, denoted PV) of the stock of the invasive, X, can be thought of as the optimal control costs today, the damages today, and then a discounted sum of the stream of control costs and damages into the future. We denote the discount factor as δ. The harvest in period t, h_t, is a function of the stock of the invasive X_t, the effort level e_t, and a random variable, ε_t.

$$h_t = h(X_t, e_t, \varepsilon_t).$$
(1)

The variable ε represents the stochastic relationship between harvests, efforts, and population size. Future populations of the invasive will depend on the post harvest population and the species specific growth function, g.

$$X_{t+1} = g(X_t - h_t). \tag{2}$$

The resource manager's objective is to maximize the discounted total expected present value of species management:

$$E\left\{\sum_{t=0}^{\infty} \delta^t \left(-c(e_t) - d(X_t)\right)\right\}, \tag{3}$$

subject to the harvest and species growth constraints (1) and (2) for all t given an initial population size $X_0>0$. Given this setup, the following functional equation characterizes the optimal solution:

$$PV(X_t) = \max_{e_t} - c(e_t) - d(X_t) + \delta E[PV(X_{t+1})], \tag{4}$$

subject to (1) and (2) given an initial population size X_0. Efficiency requires that effort should be chosen so that the marginal cost of the effort is equal to the marginal benefit from reducing the population:

$$c'(e_t) = -E[PV'(X_{t+1} \mid \varepsilon)g'(X_t - h_t)\tfrac{\partial h(X_t,e_t,\varepsilon)}{\partial e_t}]. \tag{5}$$

The left-hand side represents the marginal cost of efforts in period t. The right-hand side represents the expected marginal benefits of efforts in period t. Given ε, an increase in efforts changes harvests, which in turn influences the species growth and the species population next period. The right-hand side captures the resulting change in the expected present value of species control starting next period, where the expectation is taken over ε.

3. Uncertainty

The above problem becomes more interesting and perhaps more realistic when the population of the invasive is not known with certainty. Managers in an uncertain world are no longer able to make policy decisions based on the true population of the invasive, but rather on estimates of those populations. Understandably, as the manager's estimate becomes more accurate, the closer the optimal policy resembles the solution when the population is known.

In the presence of uncertainty, a manager no longer has a simple population of the invasive that they must deal with, but instead is forced to optimize over the belief on uncertain population, i.e., a probability distribution of the possible population size. Every potential population size should be considered when determining the appropriate effort level.

Let F be the cumulative distribution function that the manager has over the population of the invasive. That is, $F(x)=P(X{\leq}x)$. Let f be the associated probability density function. The following functional equation characterizes the manager's maximization problem:

$$PV(f_t) = \max_{e_t} - c(e_t) - \int_0^\infty f_t(x)d(x)dx + \delta E[PV(f_{t+1})].$$ (6)

Instead of the actual population, X, present values are based on the distribution of possible populations and expected damages are taken instead of actual damages. The largest change, and the largest difficulty, lies in determining what f_{t+1} is.

3.1 Bayesian Updating

Bayesian Updating is a method of determining the distribution derived from previous beliefs and observable data. In this case, the harvest that the managers observe can be used directly in our Bayesian updating framework. First, let us define:

$$\tilde{x} \equiv h_t + g^{-1}(x), \tag{7}$$

or in other words, $\tilde{x}$ is the population last period that would yield a population of x this period. Were there additional uncertainty in the growth function, $\tilde{x}$ could not be defined as a single variable, and updating would become even more difficult. Now we can write

$$f_{t+1}(x \mid h_t, e_t, f_t(x)) = \frac{\pi(h_t \mid X_{t-1} = \tilde{x}, e_t) f_t(\tilde{x})}{\int_0^\infty \pi(h_t \mid X_{t-1} = x', e_t) f_t(x') dx'}, \tag{8}$$

where π is the probability of observing a harvest of exactly h_t given population size $\tilde{x}$ and an effort level e_t.

Equations (6) and (8), above, together yield the necessary equations for maximizing the present value of the invasive under uncertainty. What may or may not be clear from the above equations is that now, since effort appears in the right hand side of equation (8), effort will not only affect the expected mean of the population estimate next period, but may also affect the spread. In any context where higher harvests for a particular population level would yield second order stochastically dominating distributions, effort becomes even more valuable than in the perfect certainty case. This extra value comes from the value of information.

3.2 A Specific Functional Form

The equations presented in the last section describe how a resource manager would optimally determine the level of effort to set in each period. In order to provide a more precise understanding of how harvest and information updating work in this framework, we will add three simplifying assumptions to our model. Throughout the rest of the paper we make three assumptions. First, we assume damages are linear in invasive population. With this assumption we can set $d(X_t) = dX_t$ for all X_t. Secondly, we assume an exponential growth rate of the invasive, allowing us to write $g(X_t - h_t)$ as $kX_t - kh_t$. Finally, and perhaps most significantly, we assume that control efforts are a binomial process, where each member of the population has an equal likelihood of being caught on a given round of treatment.

While these assumptions are not equally applicable to all invasive species, they can prove useful in a variety of settings. For example, while most managers agree that invasive species can cause massive economic damages, determining how much damage each individual member of the species causes can be a great challenge. For this reason, approximating damages as a linear function of the invasive population may be a reasonable estimate.

Also, not all species reproduce at an exponential rate. Resource economics typically assumes a logistic growth function, where the rate of growth slows as the population increases, to the point where annual growth rate declines past the point of maximum

sustainable yield. However, invasive species introduced to a new ecosystem usually have ample supply of prey-base and room to expand. The ability to rapidly multiply in number is one factor that makes invasive species such a problem in many cases, and makes an exponential growth function more reasonable.

Our most restrictive assumption is the availability of a binomial control strategy. In order for a control process to qualify it must have a repeatable methodology, with a likelihood of successful harvest of a given member in any iteration that is independent both of the harvest of other members of the population as well as independent of past control activities. This would exclude species where a large percentage of the population could be captured all at once (such as pack animals) or any control strategy that systematically cleared an area of the pest.

This approach is defendable for a surprisingly wide variety of species, even ones that may not seem applicable. One example is that of sessile species, such as any of the numerous noxious weeds that threaten agriculture. If each sweep of an area is considered one round of the binomial event, so long as there is an imperfect chance of finding a given weed on a particular round, then frequency of the sweeps may depend on expected size of the population. For example, suppose a manager attempts to rid the managed area of spotted knapweed. The manager knows that it will be difficult to get 100% of the knapweed on the first sweep of the area. The assumption is that managers can capture a higher percentage of the knapweed with either repeated sweeps or more intense sweeps at the same effectiveness per man-hour.

As for animals that could potentially be caught at one time, thus violating the independence assumption, the problem can be alleviated by changing the unit of analysis. For example, red imported fire ants are likely to live together in anthills, and thus if one fire ant is discovered, it will be likely that a whole colony will be controlled. However, if the unit of analysis is moved to controlling anthills (or queen ants) then the harvest of each of those units are more likely to be independent of each other.

Effort in this context is a function of control techniques such as the number of snake traps set, concentration of toxicants, or simply person-hours spent hunting. These techniques are able to capture, trap, or otherwise kill a percentage of the invasive population in one period. Furthermore, varying either the frequency or the intensity of these techniques allows the manager to set different percent capture rates. It is these expected percent capture rates that we will call e in this context.[2] For example, an $e = .95$ would mean that the manager expected to capture 95% of the invasives whatever the optimal combination of control techniques necessary. The actual percent of the population captured varies around e_t, but harvesting can never add to the population nor can it remove more of a species than are actually present.

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[2] We use expected capture rate as our measure of effort rather than the control technique itself, as is commonly done in resource economics (e.g., measuring effort as number of boats or person-hours). This allows the transition of population from one period to the next to simply be the previous population minus the proportion of snakes removed. Otherwise, the previous population would have to be reduced by a more complicated function of the control technique. For example, we would need to specify the precise relationship between the control technique and the number of individuals removed. In the case of snakes, for example, this would require specification of how many snakes would be removed through traps or search hours. This relationship is still necessary to specify in our case, although converting to capture rate simplifies the equation of motion. This substitution greatly facilitates computation in the case of harvest under uncertainty.

In addition to the plausibility and the analytical convenience explained above, this framework might be consistent with the decision making by the real world managers, who generally set effort levels for a given period and then simply harvest whatever that effort level yields. This setting is in contrast to many previous resource models where, even in the face of uncertainty, managers set a planned harvest level (or sometimes escapement level) and then merely observe whether or not that harvest depletes the population.

Given effort level e and population X, the probability that exactly h individuals are caught is given by:

$$\frac{X!}{h!(X-h)!}e^{h}(1-e)^{X-h}. \tag{9}$$

However, this equation only works for discrete values of h and X. Where convenient we will use the continuous approximation of the binomial distribution. As X increases in size, and therefore as it approaches a continuous variable, the distribution of h becomes:

$$h \sim N(e_{t}X_{t}, X_{t}e_{t}(1-e_{t})). \tag{10}$$

Separating harvest into its deterministic and stochastic components, we have:

$$h = e_{t}X_{t} + \varepsilon_{t}, \text{ where } \varepsilon_{t} \sim N(0, X_{t}e_{t}(1-e_{t})). \tag{11}$$

Let the expected proportion of the population controlled by a single unit of control effort be denoted alpha. If the effectiveness of each unit is independent, the expected proportion captured after n units are used is denoted:

$$e = (1-(1-\alpha)^{n}). \tag{12}$$

When characterizing the cost function, we assume that costs are linear in the control

activities that determine e. If the cost per unit of the removal technique is c, then the cost

of effort is:

$$c(e_t) = \frac{c}{\log(1-\alpha)} \log(1-e_t) \equiv \theta \log(1-e_t). \tag{13}$$

Our problem can then be characterized as:

$$PV(X_t) = \max_{e_t} - \theta \log(1-e_t) - dX_t + \delta E[PV(X_{t+1})], \tag{14}$$

$$\hat{h}_t = X_t e_t(X_t) + \varepsilon_t, \tag{15}$$

$$h_t = \max(0, \min(\hat{h}_t, X_t)), \tag{16}$$

$$X_{t+1} = kX_t - kh_t, \tag{17}$$

$$\varepsilon \sim N(0, X_t e_t(1-e_t)), \tag{18}$$

where $\hat{h}_t$ is potential harvest. Noting that this value cannot exceed current population nor

can it be negative), h_t is thus realized harvest.

A first order necessary condition for efficient management in this problem is that:

$$\frac{\theta}{1-e_t} + \delta \frac{dE[PV(X_{t+1})]}{de_t} = 0 \tag{19}$$

or

$$\frac{-\theta}{1-e_t} \approx -\delta k X_t dE[PV'(X_{t+1})] \qquad (20)^3$$

where the marginal cost of removal today must be equal to the present value of the

change in damages and effort in later periods. This condition describes optimal effort,

and thus control of an invasive species when harvesting under perfect certainty.

3.3 Harvest under Uncertainty

If the harvest technique is a binomial process and the expected percent capture rates that

efforts engenders are known, then the population can be estimated through the observed

harvest and effort rates. We use the normal approximation for the binomial as given

above wherever convenient.

If the manager has no prior knowledge about the stock of the invasive population at time

t=0, after observing effort they can calculate:

$$E[h_0] = E[e_0 X_0 + \varepsilon_0] = e_0 E[X_0]. \qquad (21)$$

Thus, after observing h and e, an unbiased estimate of X_t is given by:

$$E[X_0] = \frac{h_0}{e_0}. \qquad (22)$$

Note that equation (22) makes intuitive sense. If the manager expected to capture 50% of

the population and then 100 species are successfully caught, a reasonable estimate for the

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[3] If we use the normal approximation for the binomial distribution, the right hand side becomes

$$\delta k X_t dE[PV'(X_{t+1})] + \frac{1-2e_t}{2e(1-e)} E[\frac{PV[X_{t+1}]\varepsilon_t^2}{E[\varepsilon_t^2]} - PV[X_{t+1}]], \text{ where the term inside the}$$

expectation is not identically equal to zero because epsilon will be correlated with PV[X_t+1], but will be very close to zero.

total population would be 200. This estimate is approximately normally distributed with variance V:

$$V[E[X_0]] = V[\frac{h_0}{e_0}] = \frac{1}{e_0^2}V[h_0] = \frac{X_0(1-e_0)}{e_0} \,. \tag{23}$$

This information is only somewhat useful in its current state, as population estimates at time t can only be obtained after the harvest for that time period takes place. However, since our growth function is known, we can use data gathered in the previous period to estimate the next period's population before any harvest decision is made. We define M_t as the pre-harvest estimate of X_t such that:

$$M_1 = E[X_1] = E[kX_0 - kh_0] = k\frac{h_0}{e_0} - kh_0 = kh_0\frac{1-e_0}{e_0} \,. \tag{24}$$

The variance of this estimate is:

$$V[M_1] = V[E[kX_0 - kh_0]] = k^2V[E[X_0]] = \frac{k^2X_0(1-e_0)}{e_0} \,. \tag{25}$$

With this framework, we now have a means of deriving current population estimates from easily observable data. Unfortunately, the variance of these estimates does depend on the true stock size, an unknown parameter for the managers. In order to maintain normality and simplify our results, in our analysis we make the assumption that the managers know the variances of their estimates with certainty. Realistically, these variances would have to be estimated imprecisely, but would likely yield similar results.

Continuing with our example, we now have a pre-harvest expected population for period one. With this information, the manager is able to make a more informed decision for second period effort levels. However, after harvesting a second time, the manager could

now form a second estimate for the population at time 1 from this new information. This new estimate can be written as:

$$x_1 = E[X_1] = \frac{h_1}{e_1},$$ (26)

with variance $V[x_1] = E[X_1] = \frac{X_1(1-e_1)}{e_1}.$ (27)

Except in very rare cases, this new estimate, x, will not equal the previous estimate of X_1. Because both of these estimates are normally distributed, obtaining the distribution for the most likely estimate for X_1 is straightforward. We first define ρ_t to be the precision of the estimate M, equal to $1/V[M_t]$ and p to be the precision of our single period estimate x, equal to $1/V[x]$. Using these parameters, we can create a new estimate for X_1 that uses the available information to its fullest. As can be found in any elementary statistics book, the new estimate using both sets of information becomes:

$$\frac{M_1\rho_1 + x_1 p_1}{\rho_1 + p_1},$$ (28)

where the variance is:

$$\frac{1}{\rho_1 + p_1}.$$ (29)

Generalizing this, and applying it to the problem of interest, the manager's problem becomes:

$$PV(M_t, \rho_t) = \max_{e_t} -\theta \log(1-e_t) - dM_t + \delta E[PV(M_{t+1}, \rho_{t+1})],$$ (30)

$$\hat{h}_t = X_t e(M_t, \rho_t) + \varepsilon_t,$$ (31)

$$h_t = \max(0, \min(\hat{h}_t, X_t)),$$ (32)

$$X_{t+1} = kX_t - kh_t,$$ (33)

$$\varepsilon \sim N(0, X_t e_t (1 - e_t)),$$ (34)

$$M_{t+1} = k \frac{(M_t - h_t)\rho_t + (x_t - h_t)p_t}{\rho_t + p_t},$$ (35)

$$\rho_{t+1} = \frac{\rho_t + p_t}{k^2},$$ (36)

$$x_t = \frac{h_t}{e_t},$$ (37)

$$p_t = \frac{e_t}{X_t(1 - e_t)}.$$ (38)

where $\hat{h}_t$ and h_t are again potential and realized harvest as described in the previous

section, X_{t+1} is the new (uncertain) population, M_{t+1} is the pre-harvest estimate of X_{t+1},

ρ_{t+1} is the precision of the new pre-harvest estimate, x_t is the estimate of the population

using only harvest and effort in time t, and p_t describes the precision of this estimate.

In this uncertain framework, effort decisions can only be made based on the expected

population M and the precision with which we can make that estimate, ρ. The present

value is thus not only a function of the stock of the invasive, but also of the information

held at time t. Our first order equation now becomes:

$$\frac{\theta}{1 - e_t} + \delta \frac{dE[PV(M_{t+1}, \rho_{t+1})]}{de_t} = 0$$ (39)

or

$$\frac{-\theta}{1 - e_t} \approx -\delta k E[X_t PV_1(M_{t+1}, \rho_{t+1})] + \frac{\delta}{k^2(1 - e_t)^2} E[\frac{PV_2(M_{t+1}, \rho_{t+1})}{X_t}].$$ (40)

The term $PV_1(M_{t+1}, \rho_{t+1})$ refers to the derivative of the present value with respect to the

first argument, M_{t+1}, and $PV_2(M_{t+1}, \rho_{t+1})$ is the derivative with respect to the second term,

ρ_{t+1}. Expected values are taken both over all possible stocks of the invasive, X, and over

all possible harvest levels, h, given X. The first term on the right hand side of the

equation is equivalent to the right hand side of equation (20) and represents the marginal

benefit from reducing the population in the next period.

Effort in the context of uncertainty now gains a secondary benefit. Not only does it

reduce the expected population next period, but it also increases the precision with which

the manager knows the population. The second term on the right hand side of equation

(40) represents the additional value gained from increasing the precision of the

population estimate in the next period. The value of this term is the additional value

above and beyond the damages avoided from removing the invasive species from the

existing population, one almost always ignored in the literature to date.

4. Case Study: Brown Treesnake on the Island of Saipan

In order to illustrate the applicability and the desirability of a population updating

approach, we now turn to the case of the BTS on Saipan, located north of Guam in the

Commonwealth of the Northern Mariana Islands. BTS has already been noted to cause

extensive economic damage on Guam in the form of biodiversity losses, massive power

outages, and health costs (Savidge 1987, Fritts et al. 1987, 1990, 1994). Because of the

heavy military presence on Guam and its accompanying movement of people and cargo off island, BTS threatens surrounding islands in the Pacific. A recent estimate of an optimally managed BTS population in Hawaii, for example, approaches $18 million dollars (Burnett et al. 2006). Although Guam authorities have tried vigilantly to minimize the chance of BTS escaping to other islands, Saipan has had several sightings of BTS, and program scientists and managers believe there is an incipient population (Nathanial Hawley, personal communication).

Based on data gathered on Guam, the cost to sweep the island looking for snakes enough times to obtain an expected percent capture rate of e is: -$2,023,000*log(1-e). For a more detailed look at the derivation of this formula, see the appendix. Assuming that BTS will cause similar damages to Saipan as it might in Hawaii, we estimate d to be 121.79 per year or approximately $10.15 per month[4]. Since prey base is likely to also be similar to that of Guam, we estimate a per month growth rate of 1.0399, and we set a monthly discount rate of 1.00165, which is equivalent to an annual discount rate of 2%.

Conversations with invasive species managers suggest that there is a small incipient population of BTS on Saipan. There have been 75 credible sightings of BTS on island, leading to zero captures. While it is possible that all 75 sightings could be due to a single snake, this is highly unlikely. For the purposes of this example, we suppose that the manager estimates that there are about 200 snakes, with a standard deviation of 100 (and therefore a precision of .0001). Even if the true population is drastically different from 200, the updating process will quickly yield a better estimate.

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[4] See Burnett et al. (2006) for a detailed description of damage estimations.

In order to illustrate the effects of learning on the optimal species management, we conducted a Monte Carlo simulation and compared two strategies—one with learning (S^L) and the other without learning (S^N). With strategy S^L, the manager updates its belief on the probability distribution of the population size based on efforts and observed harvests. Strategy S^N assumes that the manager chooses efforts in each period based on the initial belief about the species distribution without updating the belief over time.

We assume the same initial belief about the distribution of population size:

$X_0 \sim N(M_0,\ 1/\rho_0)$,

where (M_0, ρ_0) is specified as in the above section. Under strategy S^N, the manager chooses

$$e_t^N = \max\left\{0, 1 - \frac{X^*}{kM_t}\right\},\ t=0,1,2,\ \dots,T, \tag{41}$$

where X^* represents the steady-state species population size in the deterministic version of the model and where

$$M_{t+1} = kM_t(1 - e_t^N),\ t=0,1,2,\ \dots,\ T, \tag{42}$$

given M_0.[5] Strategy S^N is the optimal solution for the deterministic case $(M_t \equiv X_t,\ \rho = +\infty,\ \varepsilon_t \equiv 0)$. The true population dynamics follows equations (31)-(33) where e_t is replaced by e_t^N.

Under strategy S^L, the manager chooses

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[5] Given the parameter values specified in the previous section, X^* equals 329.

$$e_t^L = \max\left\{0, 1 - \frac{X^*}{kM_t}\right\}, \ t=0,1,2, \ldots, T, \tag{43}$$

where the mean and the precision (M_t, ρ_t) are updated according to the Bayes rule as described in equations (35), (36). Though this strategy with learning is not necessarily the optimal solution to the species management problem (30)-(38), it incorporates learning in a simple and practically relevant manner.[6] Given the resource managers' information and costs constraints, a relatively simple strategy such as S^L might be more easily implemented than an optimal strategy.

We set the time horizon T to be 240 (i.e., 20 years), and computed the present values of species control under the two strategies by running 1,000 replications. Out of 1,000 replications, the present-value return with S^N exceeded the present-value return with S^L for only 41 times. The absolute value of the present-value return—the present-value total cost—without learning was more than 6 times as large as the average present-value total cost with learning. This experiment implies that the value of information through species control can be significantly large.

Figures 1 and 2 describe the population dynamics and the effort profiles in two representative trials. In each figure, the top panel describes the true species population dynamics while the bottom panel lists the time profile of efforts under the two strategies. Figure 1 represents a scenario where strategy S^N resulted in the extinction of species

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[6] The effort decision in equation (43) is an example of "constant escapement policy." When a constant escapement policy is optimal in a deterministic resource-use model, it is not necessarily optimal in the same model with uncertainty (Clark and Kirkwood 1986, Sethi et al. 2005) while it is under a certain condition (Reed 1979).

while strategy S^L caused the population size to fluctuate around the certainty-equivalent steady state $X^*=329$. Though the long-run species population is larger under learning, the present-value total cost was 8% smaller than under no learning. Figure 2 demonstrates a contrasting case where that species control without learning resulted in insufficient effort decisions and a failure to control invasion. In this case, the present-value total cost under learning was about 1/12 of the present-value cost without learning. In all trials, the fluctuations in the effort levels under learning are larger than the fluctuations without learning: under learning, updated information allows the managers to change the efforts flexibly. Such flexible updating allows the manager to achieve, on average, a larger expected present-value return.
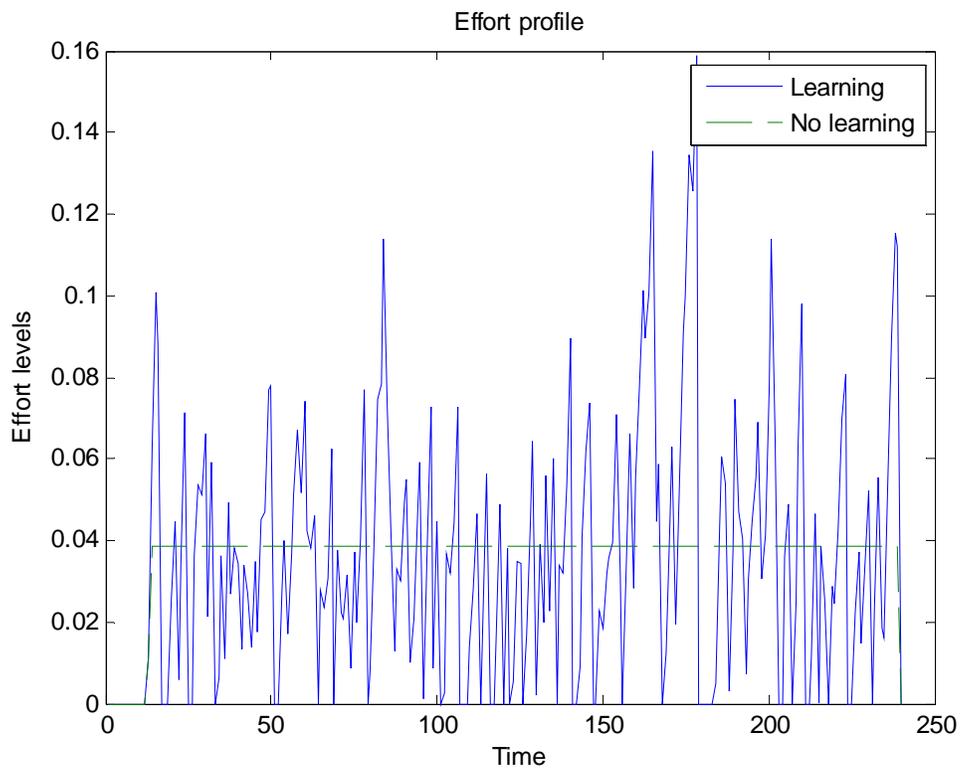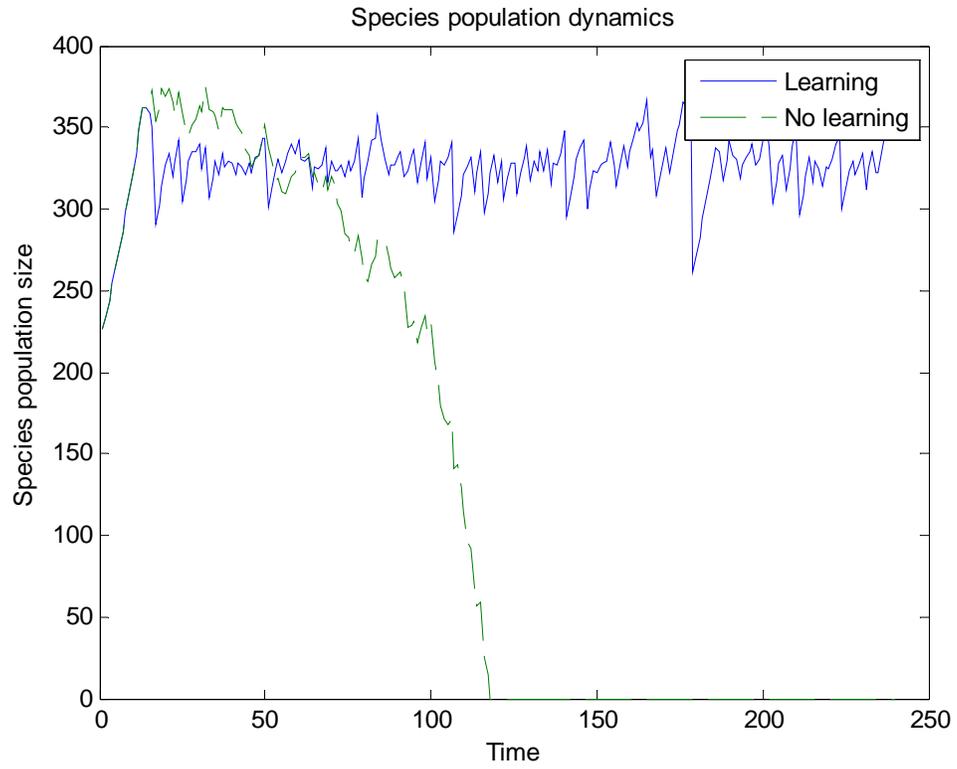
Figure 1. Species population dynamics and effort profile (I).
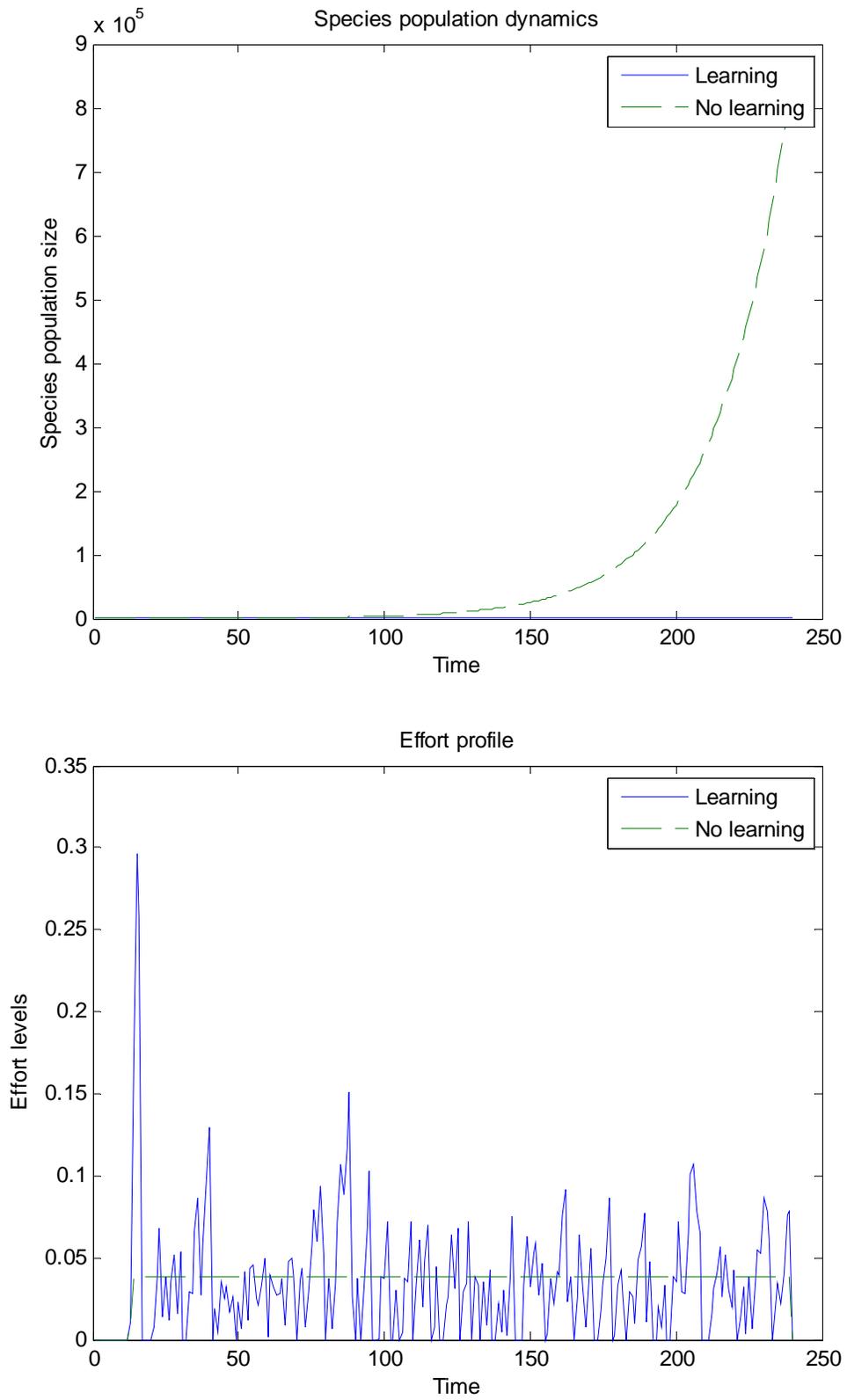
Figure 2. Species population dynamics and effort profile (II).

5. Concluding Remarks

Optimal management of an invasive species is challenging when the actual population being controlled is uncertain. However, economists and managers do have tools available to them, some relatively simple to use, to help correct for these inaccuracies. In this paper, we developed a framework to compute the optimal invasive species management where the resource manager can update their belief about uncertain species population using Bayesian methods. When effort spent on controlling the population not only lowers future expected population but also provides the manager with more precise estimates for the future, early control efforts become even more important than in models where invasive populations are known. We derived this analytical result using an example in which a binomial process is used to control the population.

By comparing the outcomes of invasive species management with and without learning, we can compute the value of additional information due to learning. In order to illustrate the application of our Bayesian model, we parameterized it to analyze BTS control in Saipan and estimate the value of information. A Monte Carlo simulation result indicates that learning reduces the cost of species control and damages in almost all cases. On average, the average cost with learning was 1/6 of the cost without learning. These results suggest that a traditional framework to analyze biological invasion given deterministic species population will not provide efficient management strategies and that value of information due to learning can be significantly large.

The model developed in this paper can easily be extended in a myriad of directions. Population estimation through stochastic harvesting processes is useful in any setting, but it is even more useful when either growth is uncertain or when the populations are subject to random shocks. While estimation techniques doubtlessly become less precise in such cases, continual updating of the population is even more necessary.

Not all resource managers have the benefit of having well calibrated control techniques where the percent of the population controlled is known. If this model were extended to include updating of the efficacy of the control techniques, then it would allow managers to learn not only about the species population, but also about their own control techniques over time. Such a model would also be necessary if members of the controlled species are heterogeneous in their catchability. In such a case, catchability of the population would naturally decline with time as the easily catchable are caught. Analysis of these issues are left for future research.

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Appendix: Cost of removal

In a recent study, Gordon Rodda of U.S. Geological Survey examined the catchability of snakes on a 5 hectare enclosed plot in Guam. Although a large degree of individual heterogeneity was found, BTS in the experiment had an average of a 16.36% chance of being caught on a given night of trapping. Saipan is approximately 12,043 hectares in

area, all of which is potential snake habitat. With the aid of Gordon Rodda, we estimate

the cost per night of trapping per 5 hectare plot to be around $150. Thus, using the

definition of θ, above, our costs of effort on Saipan will be:

$$\frac{12,043 \text{ ha.}}{5 \text{ ha./plot}} * \$150 / \text{plot} * \frac{1}{\log(.163564)} * \log(1-e) \approx -\$2,023,177 \log(1-e). \qquad (44)$$