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Abstract

We consider the prospects for sustainable growth using expected utility models of optimal investment under threat from natural disasters. Adoption of a continuous time, stochastic Ramsey growth model over an infinite time horizon permits the analysis of sustainability under uncertainty regarding adverse events, including both one-time and recurrent disasters. As appropriate to small economies, we consider adaptation to the risk of disaster. Natural disasters reduce capital stocks and disrupt the optimal consumption and felicity paths. While the time path of inter-temporal welfare might consequently shift downward, the path may still be non-decreasing over time, even without adding strong or weak sustainability constraints. Prudent disaster preparedness includes precautionary investment in productive capital, programs of adaptation to disaster risk, and avoiding distortionary policies undermining the prospects of optimality and sustainability.

JEL Classifications: O11, O44, Q20, Q28

Keywords: sustainable growth, natural disaster, expected utility, golden rule, Ramsey

1. Introduction

Linkages between poverty and the potential destruction of assets led the UN Working Group on Sustainable Development Goals to call for the integration of disaster risk management and sustainable development. While not one of the adopted 17 Sustainable Development Goals, disaster risk management is explicit or implicit in several of the targets for the achievement of goals such as poverty elimination and food security (The United Nations Office for Disaster Risk Reduction 2015). The economic theory of sustainable growth, however, is primarily concerned with conditions for optimal and sustainable growth and the theoretical foundations for measuring sustainability in the context of perfect certainty. How can the destruction of assets be incorporated into these theories?

Sustainability is an injunction to account for the interdependence between the economy and environmental resources (Stavins et al. 2003) and intergenerational equity ("not to impoverish ourselves at the expense of the future," Solow 1986). The first concern is typically modelled by putting natural capital into the production function. The second can be accommodated along the lines of Koopmans (1965), by the device of *intertemporal neutrality*, i.e. setting the planner's utility discount rate to zero. Together with feasibility conditions, this set-up provides the result that optimal capital investment and depletion of natural resources satisfies the (Arrow et al. 2004, 2012) *sustainability criterion* – that intertemporal social welfare not decrease over time – without ad hoc constraints (Ayong Le Kama 2001; Heal 2001; and Endress et al. 2005). Key to this formulation is the idea that inter-temporal welfare is the objective to be maximized, and sustainability (or non-sustainability) is a characteristic of feasible paths (Anand and Sen 2000).

These models abstract from uncertainty and show that total capital (including natural and human capitals) is increasing in the optimal (and sustainable) program. But if natural disasters are modeled as the destruction of part of the capital stock, does this mean that sustainability is a mission impossible? In what follows, we show that the optimal path can satisfy the sustainability criterion even in economies facing risk of natural disasters. As in the perfect certainty case, ad hoc sustainability conditions invoked to constrain maximization would be either redundant or counterproductive to sustaining inter-temporal welfare, where sustainability is judged according to the preservation of intertemporal welfare (Arrow et al. 2004).

The traditional literature on stochastic economic dynamics and growth is given extensive coverage by Acemoglu (2009), Adda and Cooper (2003), Gollier (2001), Sargent (1987), Seierstad (2009), Stokey and Lucas (1989), and de Hek and Roy (2001). Classic papers on stochastic growth include Brock and Mirman (1972 and 1973). In general, the literature emphasizes optimal savings in the face of uncertain income, typically arising from economic fluctuations associated with economic depressions, financial crises, or technology shocks. The prominent modeling technology in this literature is stochastic dynamic programming in discrete time.[1]

However, this strand of literature does not adequately capture how natural disasters may affect physical and ecological capital stocks. Recent work on the economics of natural disasters has made significant progress in this direction (e.g. Barro 2006; Tsur and Zemel 2006; Hallegatte and Ghil 2008; Noy 2009; Gollier and Weitzman 2010; Polasky et al. 2011; Ikefuji and Horii

[1] Our model employs continuous time stochastic optimal control, which we find more suitable for addressing the issues posed in the paper, as it affords clearer interpretation of results and avoids unnecessary computational complexity.

2012; Weitzman 2012; Pindyck and Wang 2013; Lemoine and Traeger 2016; Hallegatte 2017; Bretschger and Karydas 2018; van der Ploeg and de Zeeuw 2019). An earlier contribution is Cropper (1976). In particular, Hallegatte (2017) explores the tradeoff between security and the suppression of economic growth via excessive risk management. These contributions leave open the questions of whether and under what conditions natural disaster renders sustainable growth unattainable.

The objective of this paper is to derive insights regarding the implications of disaster risk for the prospects of long-run sustainable growth. We explore the theoretical implications of shocks in a dynamic optimization framework with intergenerational equity.[2] Examples of adverse events are highlighted to give direct relevance and application for the models presented. Consistent with Solow (1991) and Arrow et al. (2004), our analyses and results abstract from technical progress. In the interest of interpretable results, we model the probability of disaster as an exponential distribution with an exogenously specified hazard rate, P. This approach is key to answering the question of whether or not the generalized sustainability criterion can be satisfied without constraints in a world where capital stocks are subject to large and uncertain negative shocks.[3]

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[2] The model presented prominently features dynamic optimization to maximize inter-temporal welfare with inter-generational neutrality. Given that most economies operate along sub-optimal paths, well inside their production possibility frontiers due to myriad policy distortions, applying empirical data in attempt to validate or calibrate the model may be difficult to interpret.

[3] This simplification seems very reasonable for a small economy. The size of a small economy's capital stock (whether productive or natural capital) should not significantly influence the probability of a hurricane, tornado, cyclone, earthquake, tsunami, or even volcanic eruptions, to which small economies may be vulnerable. Setting P =

The paper is organized as follows. Section 2 characterizes natural disasters and surveys empirical findings on the impact of natural disasters on economic growth. Section 3 shows that the Arrow et al. (2004) criterion for sustainability is satisfied in a general Ramsey-Koopmans model of sustainable growth, wherein intergenerational equity is represented by timing neutrality. Section 4 extends the model, allowing for the stock of total capital to be subject to disaster risk, including the case of multiple disasters. Appropriate to a small economy, we also develop models of adaptation to reduce severity of disasters should they occur. Section 5 offers concluding remarks and suggestions for future research.

We find that, short of a catastrophe that completely destroys the stock of productive capital, a natural disaster does not undermine the condition that intertemporal welfare is non-declining along the optimal path. Neglect of early adaptation to disaster risk, inadequate precautionary investment, or *ad hoc* constraints on welfare maximization may lead the economy to follow a suboptimal and possibly unsustainable path with a more painful and burdensome recovery.[4]

2. The natural disaster category of adverse events

We regard natural disasters as adverse events arising from natural geologic and climatological processes of planet earth, including cyclones, flooding, tornadoes, earthquakes and subsequent tsunamis, volcanic eruptions, droughts, and wildfires. Natural disasters have the potential to result in loss of life, damage to the economy's capital stock and deterioration of

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P(K) for example, where K is the stock of total capital, would add unnecessary computational complexity to our model and render no additional insight regarding the prospect of sustaining inter-temporal welfare.

[4] We present a model of optimal adaption, cautioning that results are highly dependent on poorly defined economic geography and key parameters, such as the productivity of adaptation, with values that remain largely undetermined.

ecological systems important to sustainability and long-run social well-being. Though sourced in natural processes, these adverse events may have impacts that are highly dependent on public policies regarding risk management and disaster preparedness.

There are strong indicators, noted in news coverage and disaster policy analysis, of inadequate precautionary investment, e.g., levee failures during Hurricane Katrina and the substandard early-warning systems during the 2004 tsunami that devastated Aceh, Indonesia. Another prominent example is the compound natural disaster of 2011 in Fukushima, Japan: earthquake followed by tsunami with a consequent severe nuclear accident occurring on account of insufficient disaster preparedness at the Fukushima nuclear facility. Reckless building and development in floodplain zones, luxury homes on hurricane-hazard coasts (e.g. Hurricanes Harvey, Sandy, Irma, Maria, Florence, and Michael), the over-emphasis on channelizing streams (Katrina again), and vulnerable infrastructure (e.g. trees near power lines) constitute negative forms of precautionary capital.

The massive loss of life in the 2010 Haiti earthquake has been attributed to poor construction and building codes not being enforced (World Vision 2018). Other examples include inadequate barriers, sea walls, drainage/sewage systems, electrical power plant resilience, and preparedness for post-event response, especially in connection with vulnerable groups such as children, the elderly, and the disabled. Policy failures that can similarly reduce preparedness and resilience include subsidized insurance in hazard areas, post-event government bailouts of underinsured victims and subsequent rebuilding in coastal and floodplain zones. Ben-Shahar and Logue (2016) highlight two distortionary effects of subsidized weather insurance: (1) regressive redistribution favoring affluent homeowners in coastal communities; and (2) excessive development (and redevelopment) of storm-stricken and erosion-prone areas.

2.1. Impact on economic growth: empirical findings

Empirical evidence about the impact of natural disasters is mixed. Cavallo et al. (2010) consider a comprehensive dataset of 196 countries covering the period 1970-2008. A total of 6,530 adverse events were recorded, of which 47.4 percent were floods, 40.1 percent were storms and 12.5 percent were earthquakes. With the exception of two cases where radical political revolution followed major natural disaster, the empirical analysis showed that even large disasters did not produce significant effects on long run economic growth, consistent with neoclassical growth theory.

Noy (2009) concludes that natural disasters have a statistically observable negative impact on output in the short-run and finds that countries with a higher literacy rate, better institutions, higher per capital income, a higher degree of openness to trade and higher levels of government spending are more capable of withstanding initial disaster shocks and preventing further spillover into the economy. The literature review by Cavallo and Noy (2010) examines the short and long run effects of disasters, providing detailed accounting of output dynamics.

Laframboise and Loko (2012) consider the cases of seven countries studied by the International Monetary Fund and find a wide range of impacts and costs. Of the seven countries studied - Haiti, Japan, Kenya, New Zealand, Pakistan, Samoa, and St. Lucia - impact on output in the short run was largest for Haiti and St. Lucia while modest for New Zealand. This presumably reflects a positive relationship between per capita income and precautionary capital. The impact of disasters on long- run economic growth in these countries is not profiled.

Two more recent review articles also highlight ambiguities and gaps in the research literature. Shabnam (2014) provides a literature review of economic growth theories and their implications for addressing potential impacts of natural disasters. He finds the extant literature inconclusive: positive effects of natural disasters on economic growth in some cases and suggestions of negative or no effect in others.

Kousky (2014) reviews the economic costs of natural disaster, finding that despite high damage costs, many natural disaster events have a relatively modest impact on output and growth and that these impacts disappear fairly quickly. Some of the studies reviewed did examine how areas with different risk levels have invested differentially in hazard mitigation. Such studies, however, failed to account for timing of adaptation or separate measures that can be done in the short-term versus those that can only occur over a longer time span.

The Centre for Research on the Epidemiology of Disaster (CRED) report (2019), Natural Disasters 2018, offers striking summary data. In 2018, there were 315 natural disaster events recorded with 11,804 deaths, over 68 million people affected, and US$131.7 billion in economic losses around the world. The burden was not shared equally, as Asia suffered the highest impact and accounted for 45% of disaster events, 80% of deaths, and 76% of people affected. Flooding affected the highest number of people, accounting for 50% of the total affected, followed by storms, which accounts for 28%. The CRED report remarks that given Asia's large land mass, higher population relative to other continents, and multiple hazard risks, the results are not surprising.

In summary, the evidence regarding the effect of disasters on economic growth is mixed, possibly due to offsetting theoretical forces. Neoclassical growth theory says that capital-destroying disasters increase growth rates in the (long) convergence towards the *same* balanced-growth steady state that existed before the disaster.[5] With *creative destruction* and Keynesian unemployment, the effects could be even greater. But the possible loss of (invisible) social and

[5] See sections 3.2 and 4.1 for discussions of convergence.

institutional capital and the disrupting impacts on expectations and investment may offset these. Sorting out these effects is a challenge for future research but will presumably show that the net effect differs depending on the individual cases examined.

2.2. Other prospects for empirical research

As we show in section 4.1.2, the higher the percentage of capital stock damaged by natural disaster (D), the greater the time T of recovery to re-attainment of the pre-existing capital stock. Inadequate precautionary investment tends to yield sub-optimal growth with increased pain and loss of life in the short run and longer recovery times. This is difficult to account for empirically, however.

Another factor not fully addressed in the empirical analysis (perhaps due to its lack of prominence in theory) is the potential to push natural capital below critical level tipping points, undermining prospects for long run sustainability.[6] This potential may have particular relevance for coastal areas. Without careful adaptation design, including rezoning and coastal flood inlets in addition to structures such as seawalls and groynes to absorb wave energy, flooding from major storms and king tides may irreversibly erode beaches, coral reefs, and coastal areas. Fierce hurricanes, cyclones, typhoons, and fires can create lasting damage to critical watersheds, while flooding and sea level rise can lead to major salt-water intrusion into fresh-water systems, potentially reaching overwhelming proportions.

3. Modeling sustainable development with no risk of disaster

[6] For theoretical treatments of tipping points and regime shifts see, for example, Polasky et al. (2011), Lemoine and Trager (2016), and van der Ploeg and de Zeeuw (2019).

The oft-quoted Brundtland definition of sustainable development about meeting the needs of the present without compromising the opportunities of the future (World Commission on Environment and Development, 1987) conveys a sense of balancing the needs of the present generation with stewardship for the future. A central parameter of the balancing act is called the *pure rate of time preference*, ρ, the rate at which social preferences discount the welfare of future generations. Inasmuch as a positive rate of time preference is tantamount to discrimination against the future (Heal 2000, 2001), we consider the case of ethical neutrality (Koopmans 1965), i.e. setting the planner's utility discount rate to zero.[7]

3.1. Conventional Ramsey growth model with intertemporal neutrality

We start with a growth model, formulated in terms of total capital stock K in the economy.[8] We use felicity, U(C(t)), as a measure of economic well-being at a particular time t.[9] A typical functional form is $U\big(C(t)\big) = -C(t)^{-(\eta-1)}$, for $\eta > 1$, where the consumption elasticity of marginal felicity, $C[U''(C)]/U'(C)$, is equal to -η. The aggregate production function for the economy is specified as $F(K(t)) = A[K(t)]^{\alpha}$, where A represents productivity in the economy and α is the output elasticity of capital. Labor in the economy is taken as constant and does not enter as an argument in the production function.[10] The total capital stock depreciates at constant rate δ.

[7] Ramsey, Pigou, Sidwick, Harrod, Koopmans, Stern, and others have decried the immorality of discounting the utilities of future generations. See Arrow (1999) and Endress et al. (2014) for an inventory of quotations.

[8] Following Weitzman (1976), K can be viewed as a composite of produced, natural, and human capital, aggregated from the shadow prices of the components.

[9] "Felicity" is the name given to the auxiliary function U in order to distinguish it from the social planner's utility function (e.g. Dasgupta and Heal 1979 and Arrow et al. 2012).

[10] This is easily generalized to the case where population grows at a constant rate (Solow 1956).

The dynamic feasibility constraint for the economy, $\dot{K} = F\big(K(t)\big) - \delta K(t) - C(t)$, specifies investment in new capital, $\dot{K}$, as the residual output after subtracting aggregate consumption, C(t), and replenishment of capital depreciation, δK(t). A steady state for the economy may eventually be attained if the output elasticity of capital, α, satisfies α < 1, and the stock of natural capital satisfies certain conditions. A steady state for the economy is characterized by the condition that the growth rate $g = \dot{C}/C(t) = 0$, so that $F_K = \delta$. Let K* designate the steady state level of the total capital stock, for which the marginal product of capital equals the rate of capital depreciation. With this steady state level of total capital, $\dot{K} = \dot{C} = 0$, and we can write steady state aggregate consumption as C* = F(K*) - δK*. This steady state condition comprises the classic Golden Rule associated with the standard Solow growth model. Golden Rule felicity, U(C*), is then used to formulate the dynamic optimization problem.

As shown in Koopmans (1965), felicity can be indexed as the gap between U(C(t)) and U(C*) in order to avoid the problem of a non-convergent objective function. As the economy grows, both C(t) and U(C(t)) increase, and the gap, {U(C(t)) – U(C*)}, narrows, becoming less negative.

We then consider the undiscounted flow of intertemporal welfare from time t forward, computed as the integral $\int_t^\infty \{U(C(s)) - U(C^*)\}ds$. The planner's problem is to choose the consumption trajectory C from t to ∞ that maximizes undiscounted intertemporal welfare. We denote the maximized value of intertemporal welfare as:

$$V(t) = \max_C \int_t^\infty \{U(C(s)) - U(C^*)\}ds \tag{1}$$

subject to $\dot{K} = F\big(K(t)\big) - \delta K(t) - C(t)$, for all $s \geq t$. For initial time $t = 0$, $V = V(0)$.

This approach, with the integrand specified as {U(C(s)) – U(C*)}, is the *Koopmans transformation*, based on the theory presented in Koopmans (1965).[11,12] With consumption C as the control variable and K the state variable, the problem is easily solved using the maximum principle in optimal control theory. The dynamic first order condition for the optimal consumption path is Ramsey's (1928) optimal saving equation with $\rho = 0$:

$$r = F_K - \delta = \eta g \tag{2}$$

The letter r designates the (capital) rate of return (Barro and Sala-i-Martin, 2004) and is also referred to as the discount rate (Gollier, 2013). A key consequence of following the optimal path is that intertemporal welfare is non-declining over time; that is $\dot{V} \geq 0$, thereby satisfying the Arrow et al. (2004) condition for sustainability. The steady state is characterized by $\dot{K} = 0$ and g = 0 in equation (2). The economy approaches the steady state as t approaches infinity; U(C(t)) approaches U(C*), and V(t) approaches zero from below. No sustainability constraints are required to achieve optimality.

3.2. Convergence to the steady state with no risk of disaster

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[11] Without the Koopmans transformation, there can be many consumption paths for which the integral of U(C) is infinite (does not converge). By this means and by restricting attention to paths that are both feasible and "eligible" (undominated), Koopmans (1965) established that the transformed welfare function provides a complete ranking of those paths.

[12] One alternative to ethical neutrality via the Koopmans transformation is the von Weizsacker (1965) overtaking criterion. This criterion and modeling technique are used by Brock and Mirman (1973) for the case of stochastic optimal growth with no discounting. Another approach to modeling sustainable social preferences has been suggested by Chichilnisky (1996). In place of either the Koopmans or the von Weizsacker approach, she posits axioms specifying no "dictatorship" by either the present or the future.

A closed form solution for the path of K(t) from initial time zero to the steady state is typically formulated using approximation techniques, such as the process of log-linearization applied to the Ramsey equation. The process involves taking logs of key variables followed by performing a first order Taylor series expansion around steady state variables. The detailed process is presented in Barro and Sala-i-Martin (2004), Section 2.6.6 (Speeds of Convergence, pp 111- 118) and Section 2.8 (Appendix 2A: Log-Linearization of the Ramsey Model, pp 132-134). The result is an approximate path of convergence of log[K(t)] to the steady state value log[K*]:

$$\log[K(t)] = [1 - e^{-\sigma t}] \log[K^*] + [e^{-\sigma t}] \log[K_0] \tag{3}$$

In accordance with the exponential specification, the path rises asymptotically to the steady state value log[K*], exhibiting a concave profile.

The parameter σ is known as the speed of convergence; its value is determined by the parameters of preferences (η, ρ) and production (α, δ, but surprisingly not A, which produces opposite, offsetting effects); however, it does not depend on K_0, the initial stock of capital. Barro and Sala-i-Martin (2004) mention (p 112) that their numerical approximation of σ = 0.02 per year, corresponding to α = 3/4, closely matches empirical estimates. The specific formula for computing σ is given by equation 2.41 of Barro and Sala-i-Martin (2004). Assuming ρ = 0, no technological change, and no population growth, this formula reduces to σ = (1 - α)(δ)/√(αη). With these simplifying assumptions on parameters in the formula, a speed of convergence σ = 0.02 for α = 3/4 implies that η = 2. Note that for these typical values, √(αη) = 1.22 ≈ 1.

The formula for speed of convergence has a straightforward interpretation. The rate of capital depreciation δ determines the rate at which capital needs to be replenished along the optimal path. The greater the rate of needed replenishment, the greater the amount of investment in the economy, rendering faster adjustment. On the other hand, the higher the output elasticity of capital α, the closer the economy is to linear production (AK), slowing convergence to the steady state. In the extreme case of AK production, there is no convergence. High values of η indicate a preference for consumption smoothing and social aversion to inter-temporal inequality, resulting in flatter growth paths and slower approaches to the steady state. Accordingly, speeds of convergence are reduced. In the limit as η approaches infinity, the speed of convergence goes to zero and the economy exhibits a flat path of constant, maxi-min consumption. (See Dasgupta and Heal (1979), chapter 10, for discussions of the parameter η and its effect on growth paths.) An analogous formula for speed of convergence in a basic Solow growth model reduces to $\sigma = (1 - \alpha)(\delta)$. (In addition to Barro and Sala-i-Martin (2004), chapter 1, see Acemoglu (2009), p 81.)

The "half-life of convergence" ($T_{1/2}$) is the time interval required to close the gap between $\log[K_0]$ and $\log[K^*]$ by one half. Algebraic manipulation of equation (3) shows that $T_{1/2} = [\log 2]/\sigma$; for $\sigma = 0.02$, $T_{1/2} = 0.693/0.02 = 34.66$. So with time measured in years, the half-life of convergence is about 35 years. To close the gap again by half takes another 35 years and so on. To close the gap between $\log[K_0]$ and $\log[K^*]$ 7/8 of the way, for example, would then take 3(35) = 105 years.

4. A stochastic Ramsey growth model with intertemporal neutrality

Following Tsur and Zemel (2006), we model the probability of natural disaster as an exponential distribution. We let parameter P represent the hazard rate of an abrupt occurrence of a natural disaster, such that Pdt measures the conditional probability that the event will occur during $[t, t + dt]$, given that it has not occurred by time t.[13] Let T represent the random event-occurrence time with cumulative probability distribution, F(t), and probability density function, f(t). The hazard rate is related to F(t) according to $P = f(t)/(1 - F(t))$, such that $F(t) = 1 - e^{-Pt}$ and $f(t) = F'(t) = Pe^{-Pt}$.

We model the adverse impact of a natural disaster as a loss of fraction D_K of the capital stock K. Short of a total catastrophe ($D_K = 1$), disaster recovery is possible through the process of capital replacement and restoration as we subsequently show. Along with an exogenous hazard rate P, the fraction D_K is taken as exogenous.[14]

Following Ikefuji and Horii (2012), we regard disaster risk as an additive, stochastic augmentation to capital depreciation. Pindyck and Wang (2013) do something similar using Poisson arrival times and a Brownian motion process to model capital shocks. Employing the Koopmans (1965) transformation used in section 3, the planner's problem is to choose the consumption path C to maximize the planner's expected utility. We denote the maximized value of the planner's expected utility as:

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[13] The hazard rate in Tsur and Zemel (2006) is a function of the environmental stock. In the present treatment, the hazard rate is a parameter.

[14] Treating damage fraction D as a constant proportion of the capital stock is assumed to be a reasonable approach in the basic disaster model (without adaptation). We first regard all capital in the small economy as facing the same risk of natural disaster. Increasing the capital stock through the process of accumulation adds to the total stock of capital at risk to the same degree. In Section 4.3, we endogenize D_K in a model of optimal investment in adaptation and derive a minimized damage fraction D*. We do not consider negative shocks to productivity, which van der Ploeg and de Zeeuw (2018) regard as regime shifting, and possibly non-recoverable, disasters. They address productivity shocks by permanently reducing total factor productivity A by an exogenous factor $(1 - \pi)$ with $(0 < \pi < 1)$ post disaster.

$$W(t) = \max_{C} E\left(\int_{t}^{\infty} \{U(C(s)) - U(C^{*})\} ds\right) \tag{4}$$

subject to $\dot{K} = F(K) - \{\delta + Pe^{-Pt}(D_K)\}K - C$, where E is the expectations operator.

4.1. Solving the planner's problem

With the prospect of natural disaster and the risk of damage-induced jumps in state and costate variables, application of the standard maximum principle for a closed-form, analytical solution to the optimal control problem is questionable, but may be adaptable as a first stage to a numerical approximation of disaster recovery, discussed below. In that regard, we emphasize that satisfying the sustainability criterion of non-declining intertemporal welfare is not dependent on post-disaster, numerical approximation in the form of log-linearization.

Even in the case of non-stochastic jumps, either planned or anticipated by the planner at fixed times, required adjustments to the maximum principle are non-trivial as described in Arrow and Kurz (1970) and Seierstad and Sydsaeter (1987). More complex yet are the cases of uncertainty where jump times τ arrive randomly. Stochastic dynamic programming is a prominent approach for managing these cases. Tsur and Zemel (2006) use that approach to derive the welfare implications of a single catastrophic event. Seierstad (2009), chapter 3, considers the possibility of multiple jumps, finite in number, at stochastic points in time, again using the technology of stochastic dynamic programming. The development though is limited mainly to finite time horizons, without evident application to the economics of sustainable growth over infinite time horizons.

Yet, a useful first-stage strategy is described for finite time horizons in Seierstad (2009), Section 3.2, and characterized as a 'standard elementary method' to facilitate solution: first solve the optimal control problem using the basic maximum principle but as if actual jump events do not occur within the finite time period of the problem. We adapt this strategy to the case of an infinite time horizon by regarding actual occurrences of natural disaster as being postponed indefinitely. This first stage computation yields what might be considered a 'hypothetical' jump-free path, not actually realized, but yet serving as a productive basis for approximating a solution. In the next stage, Seierstad (2009) augments the maximized Hamiltonian of the first stage by adding and applying 'extremals' or 'characteristic equations' associated with the Hamilton-Jacobi-Bellmen (HJB) equations of stochastic dynamic programming. This subsequent stage then relies on backward recursion, starting with the last of an assumed finite number of random events.

For the next stages of approximation, we continue with the framework of optimal control over an infinite time horizon, rather than resorting to stochastic dynamic programming. A major advantage to the optimal control setting is applicability of, and access to, the technique of log-linear approximation, derived from ordinary differential equations. In contrast, dynamic programming yields HJB partial differential equations, not readily conducive to the same method of log-linear approximation as discussed in Section 3.2.

The first stage 'hypothetical' optimal control solution via the basic maximum principle yields an extended Ramsey equation including disaster risk:[15]

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[15] The planner first solves the problem (equation (4)) assuming disaster risk, but as if actual occurrence were postponed indefinitely (the details of the optimal control solution for the planner's problem are available in the appendix). Under that initial assumption, the extended Ramsey condition, equation (5), applies for all time t. Then, when an individual disaster does occur, say at time $t = \tau_1$, the planner adjusts according to a post-disaster numerical approximation of capital restoration (equation (7) and figure 1).

$$r = F_K - \delta = \eta g + Pe^{-Pt}(D_K) \quad (5)$$

Similar to the terminology in Gollier (2013), r is the discount rate along the optimal path in an economy facing uncertainty, in this case, risk of natural disaster. The second term on the RHS of (5) represents the precautionary effect, requiring investment in precautionary capital. For the usual case with $\eta > 1$, the precautionary effect is positive.[16]

4.1.1 The central issue: non-declining intertemporal welfare

For an individual disaster, with or without subsequent risk, the optimal path would follow a two-part Ramsey equation. Before the disaster occurs ($t < \tau_1$), equation (5) applies; the extended Ramsey equation accounts for the possible risk of natural disaster. After disaster occurs, a modified version of the extended Ramsey equation, with a time reset to $(t - \tau_1)$, now applies until the next disaster occurs:

$$r = F_K - \delta = \eta g + Pe^{-P(t-\tau_1)}(D_K) \quad (6)$$

For the special case of a single, one-time-only disaster with no subsequent risk, we can set $P = 0$ in (6) above, reducing the expression to the conventional Ramsey equation (2) for $t > \tau_1$. For the

[16] There are opposing forces in precautionary investment. On the one hand, a greater hazard rate lowers the expected marginal product of new capital, tending to lower investment. On the other hand, there is a need for greater investment to increase consumption in the event of disaster. Specifically, for $\eta > 1$, precautionary investment increases with parametrically increasing P. That's also the case in our present model as reflected in equation (5). For $\eta < 1$, there is less precautionary investment and more consumption now before disaster strikes. For $\eta = 1$ [$U(C) = \log C$], the opposing forces are exactly offsetting; an increasing hazard rate has no effect on the level of precautionary investment. These results can be established using an adaptation of the two-period model in Gollier (2013) along with the more general felicity function $U(C(t)) = \frac{[C(t)^{(1-\eta)} - 1]}{1-\eta}$ for $\eta > 0$.

period of disaster recovery from time τ_1 to $\tau_1 + T$, equation (7) is used to approximate the path for restoration of damaged capital. We regard this restoration of capital as being accounted for in the Ramsey equation over the recovery period, particularly in the term F_K. Both equation (7) and estimates for recovery time T are derived in section 4.1.2. We emphasize that satisfying the sustainability criterion of non-declining intertemporal welfare is not dependent on post-disaster, numerical approximation. In section 4.2, we consider the case of recurring and possibly overlapping disasters.

Figure 1 depicts the paths of U(C(t)) and W(t) in the disaster scenario. In the upper graph, U(C(t)) rises along the optimal path from t = 0 to t = τ_1, at which time a natural disaster occurs. The capital stock drops from $K(\tau_1)$ to $(1 - D_K)K(\tau_1)$, resulting in a discontinuous drop in optimal consumption and felicity at time τ_1. Recovery takes place over the time interval T, during which optimal consumption is lowered to accommodate the rebuilding of capital. Nonetheless, the growth path for U(C) continues to rise and still exhibits a concave profile given that $U''(C) < 0$. After recovery, U(C) continues the asymptotic approach to U(C*). Of note, the path of U(C) with the drop at time τ_1 resulting from disaster, enlarges the negative area between U(C) and U(C*) over the full time horizon. In addition, precautionary savings in the risk economy reduces consumption at each time t, further lowering the path U(C(t)). By comparison, the higher and lighter dashed curve in the upper graph depicts the path of felicity $U^{\#}(C(t))$ for the no-risk economy without precautionary savings or occurrence of disaster. Consequently, the time path for intertemporal welfare W shifts downward compared to the path V for a risk free economy. The lower graph in Figure 1 shows the two trajectories of intertemporal welfare, with the higher and lighter, dashed curve V(t) for the no-risk economy, and the lower, darker curve W(t), for the disaster scenario. The gap between V(t) and W(t) is widest at t = 0 because the difference in total areas associated with the respective integrals gets smaller as t increases.

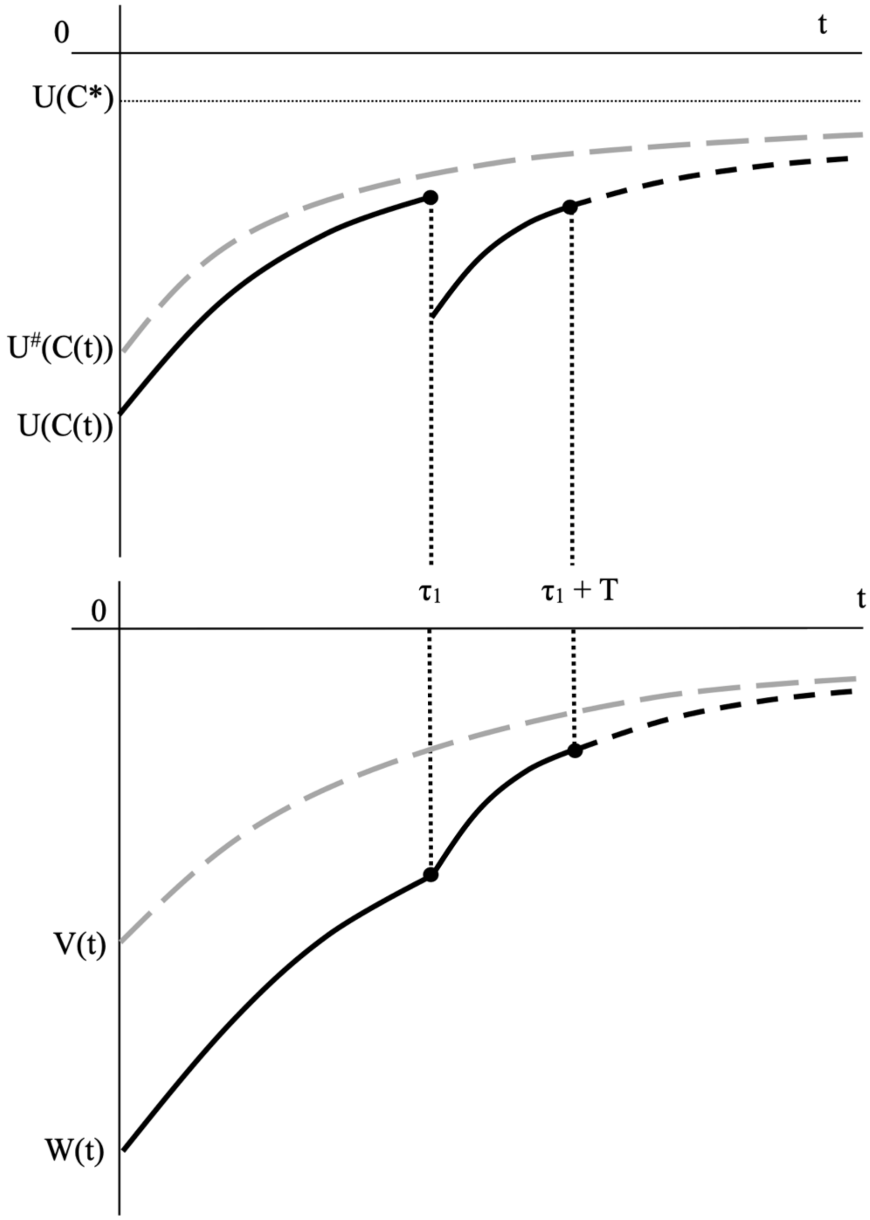

Fig. 1 Trajectories of felicity, U(C(t)), and intertemporal welfare, W(t). Disaster occurs at time τ_1. Duration of recovery is T. The dashed portions of the darker curves signal the possibility of subsequent disasters should the economy continue to face disaster risk. The higher and lighter dashed curves for $U^{\#}(C(t))$ and V(t) in the two graphs depict paths for the contrasting no-risk economy.

Even with the jump discontinuity in U(C(t)) at time t = τ_1, {U(C(t)) – U(C*)} remains an integrable function. Hence, in line with the fundamental theorem of calculus, the indefinite integral W(t) is continuous, but differentiable only at points where {U(C(t)) – U(C*)} is continuous.[17] Accordingly, W(t) fails to be differentiable at t = τ_1. But note that [U(C*) – U(C)] is positive at both $t = \tau_1^-$ and $t = \tau_1^+$ despite the jump drop; hence, W is still increasing at points to the immediate left and right of the drop, thereby satisfying the sustainability criterion. In other words, based on equation (4), with C* constant and greater than C(t) for all finite t, a straightforward result where W(t) is differentiable is that, $\dot{W}(t) = [U(C^*) - U(C(t))] > 0$. (This result is an application of the fundamental theorem of calculus to a convergent integral of a piecewise continuous function over an infinite time horizon.) Note also that $\dot{W}(\tau_1^-) = [U(C^*) - U(C(\tau_1^-))] < [U(C^*) - U(C(\tau_1^+))] = \dot{W}(\tau_1^+)$. Since W(t) is non-declining (pre, post, and at disaster), the Arrow et al. (2004) criterion for sustainability is satisfied. We also note that the path W(t) is concave before and after the disaster: $\ddot{W}(t) = -[U'\big(C(t)\big)]\dot{C} < 0$. These results are a consequence of continuously rising paths of capital and felicity during optimal recovery.

As damaged capital is restored over the recovery period, felicity of consumption increases along a concave path from $U(C(\tau_1^+))$ to $U(C(\tau_1 + T))$, the latter being near, but not necessarily equal to $U(C(\tau_1^-))$. The project of capital restoration is funded by sacrificing some extra consumption over the recovery period, which may reduce $U(C(\tau_1 + T))$. In addition, the relative magnitudes of stochastic depreciation at times τ_1 and $(\tau_1 + T)$ will influence the relationship of the two felicities. For the case of a single disaster with no subsequent risk, there is

[17] See e.g. Friedman (1999), chapter 4, section 5 (The Fundamental Theorem of Calculus), page 127, Theorem 1: An indefinite integral of an integrable function is a continuous function.

no stochastic depreciation and associated precautionary investment for $t > \tau_1$, perhaps giving an edge to $U(C(\tau_1 + T))$. The upper panel of Figure 1 is drawn for the case $U\big(C(\tau_1^-)\big) > U(C(\tau_1 + T))$.

In summary, disruption of optimal consumption and felicity paths due to natural disaster might widen the gap between the constant steady state path U(C*) and U(C(t)) for $t > \tau_1$. As a consequence, the time path of inter-temporal welfare might consequently shift downward. Nonetheless, short of catastrophe that precludes recovery, the time path remains non-decreasing, thereby satisfying the sustainability criterion. Prudent disaster preparedness includes precautionary investment in productive capital, programs of adaptation to disaster risk, and avoiding distortionary policies undermining the prospects of optimality and sustainability.

4.1.2 Paths of convergence and recovery times

Building on the log-linearization approach discussed in section 3.2, we now account for stochastic depreciation. Taking the worst case, without exponential drop over time, total depreciation becomes $[\delta + PD_K]$, leading to the modified formula $\sigma = (1 - \alpha)[\delta + PD_K]/\sqrt{(\alpha\eta)}$. The inclusion of stochastic depreciation in the formula reflects the incentive for precautionary investment, increasing the speed of convergence in the economy facing disaster risk. With parameters $\alpha = 0.75$, $\eta = 2$, and $\delta = 0.1$, a small economy facing risk of natural disaster with hazard rate $P = 0.10$ and severity $D_K = 0.25$ would exhibit a speed of convergence $\sigma = 0.0255$ per year, yielding a half-life of convergence $T_{1/2} = [\log 2]/\sigma = 27.18$ years. To simplify notation in subsequent derivations and formulas, we set $\Phi = (1 - \alpha)/\sqrt{(\alpha\eta)}$ so that $\sigma = \Phi[\delta + PD_K]$. This formula is appropriate to an economy facing disaster risk before disaster actually strikes. After a disaster event and during the period of recovery with restoration of damaged capital, the formula for speed of convergence needs to be adjusted as we discuss below.

Suppose that a disaster occurs at time τ_1, $0 < \tau_1 < \infty$, destroying fraction D_K of accumulated capital stock $K(\tau_1)$. Because of precautionary investment, $K(\tau_1)$ under uncertainty exceeds the level $K(\tau_1)$ that would apply in the case of certainty and no risk of disaster. Capital remaining is $(1 - D_K)K(\tau_1)$, which functions as the initial capital stock for the trajectory from time τ_1 forward. The post-disaster trajectory of capital is characterized as follows:

$$\log[K(t-\tau_1)] = \left[1 - e^{-\sigma(t-\tau_1)}\right]\log[K^*] + \left[e^{-\sigma(t-\tau_1)}\right]\log[(1-D)K(\tau_1)] \qquad (7)$$

In spite of disaster occurring at time τ_1, the value for K^* remains unchanged, since it depends on the fundamental parameters of the economy; steady state capital K^* is determined by A, α, δ (but not K_0), where we have assumed $\rho = 0$ and no productivity shocks that would reduce A. Equation (7) represents a reset and restart of the economy from time of disaster τ_1 with $[(1 - D_K)K(\tau_1)]$ in place of K_0 serving as the "initial" capital stock.

With respect to resilience and recovery, we can estimate the time interval $T = (t - \tau_1)$ required to restore the capital stock back to $K(\tau_1)$ from $[(1 - D_K)K(\tau_1)]$. Substituting $\log [K(\tau_1)]$ on the left-hand side of equation (7) and solving for $(t - \tau_1) = T$ yields

$T = (1/\boldsymbol{\sigma})\log\{1 + [\log(1 - D_K)]/[\log(K(\tau_1)/K^*)]\} = (1/\boldsymbol{\sigma})\Omega[D_K, (K(\tau_1)/K^*)]$,

where $\Omega = \Omega[(D_K), (K(\tau_1)/K^*)] = \log\{1 + [\log(1 - D_K)]/[\log(K(\tau_1)/K^*)]\}$.

Robust recovery is contingent on the economy's ability to follow the efficient path approximated by equation (7) with the speed of convergence, now designated $\boldsymbol{\sigma}$, adjusted to account for restoration of damaged capital during the recovery period.

The approach we adopt for adjusting speed of convergence after disaster strikes is to expand the expression for total depreciation in the formula by accounting for restoration of damaged capital $(D_K)[K(\tau_1)]$ over recovery period T using an average proportional rate $(D_K)/T$: $\boldsymbol{\sigma} = \Phi[(\delta + PD_K) + (D_K)/T]$. We retain the worst-case stochastic depreciation term PD_K to account for the risk of a second, overlapping disaster during recovery from the first. As mentioned earlier, the special case of a single, one-time-only disaster with no subsequent risk can be represented by setting $P = 0$.

Combining two simple expressions involving T and $\boldsymbol{\sigma}$, we can solve for T:

$$T = (1/\boldsymbol{\sigma})\ \Omega = \Omega(1/\{\Phi[(\delta + PD_K) + (D_K)/T]\}$$

$$= \Omega T/\{\Phi[(\delta + PD_K)T + (D_K)]\}, \text{ which reduces to}$$

$$T = [\Omega - (D_K)\Phi]/[\Phi(\delta + PD_K)].$$

Substituting for T in the expanded formula for $\boldsymbol{\sigma}$ and simplifying, yields

$$\boldsymbol{\sigma} = [\Omega\Phi[(\delta + PD_K)]/[\Omega - (D_K)\Phi].$$

For comparison with the speed of convergence before disaster strikes, we observe that

$$\sigma = \Phi[\delta + PD_K], \text{ so that we may take } \Phi = \sigma/[\delta + PD_K].$$

Then substituting for Φ in the numerator of the expression for $\boldsymbol{\sigma}$, we obtain

$$\boldsymbol{\sigma} = \sigma\{\Omega/[\Omega - (D_K)\Phi]] > \sigma.$$

The speed of convergence increases after disaster strikes.

For illustration, Table 1 presents alternative calculated values of speeds of convergence σ and $\boldsymbol{\sigma}$, half-lives $T_{1/2}$, and recovery times T for different assumed values of α, (D_K), and $\Omega = \Omega[(D_K), (K(\tau_1)/K^*)]$ taking $\eta = 2$, $\delta = 0.1$, and hazard rate $P = 0.1$.

	σ (per year)	$T_{1/2}$ (years)	$\boldsymbol{\sigma}$ (per year)	T (years)
$\alpha = 1/2$				
$D_K = 1/4$	0.06	11.55		
$\Omega(1/4,1/2) = 0.347$			0.094	3.697
$\Omega(1/4,3/4) = 0.693$			0.073	9.466
$D_K = 1/2$	0.075	9.24		
$\Omega(1/2,1/2) = 0.693$			0.117	5.906
$\Omega(1/2,3/4) = 1.230$			0.0941	16.28
$\alpha = 3/4$				
$D_K = 1/4$	0.0255	27.18		
$\Omega(1/4,1/2) = 0.347$			0.0299	11.57
$\Omega(1/4,3/4) = 0.693$			0.0273	25.168

$D_K = 1/2$	0.0306	22.65		
$\Omega(1/2,1/2) = 0.693$			0.0359	19.32
$\Omega(1/2,3/4) = 1.230$			0.0334	44.10

Table 1. Comparative Speeds of Convergence. Calculations based on the formulas $\Phi = (1-\alpha)/\sqrt{\alpha\eta}$; $\sigma = \Phi(\delta + PD_K)$; $\Omega = \log\{1 + [\log(1 - D_K)]/[\log(K(\tau_1)/K^*)]\}$; $\boldsymbol{\sigma} = [\Phi\Omega[(\delta + PD_K)]/[\Omega - \Phi D_K] = \sigma\Omega/[\Omega - \Phi D_K]$; $T_{1/2} = \log2/\sigma$; $T = \Omega/\boldsymbol{\sigma}$; with $\delta = 0.1$, $\eta = 2$, $P = 0.1$.

The calculated values for σ, $\boldsymbol{\sigma}$, $T_{1/2}$ and T in Table 1 are consistent with the interpretation of formulas presented earlier. Holding other parameters fixed, the larger value of α results in a lower speed of convergence σ, and correspondingly, a longer half-life of convergence $T_{1/2}$. In contrast, a larger stochastic depreciation PD_K (with $P = 0.1$) yields a larger value of σ and hence, a shorter $T_{1/2}$. After disaster, holding other parameters fixed, a larger damage fraction D_K increases the speed of convergence $\boldsymbol{\sigma}$, but also T, since a larger gap has to be closed to attain recovery. Table 1 also shows that recovery time T from disaster depends on the ratio $(K(\tau_1)/K^*)$. The larger $(K(\tau_1)/K^*)$ represents an economy closer to its steady state at time of disaster, which thereby experiences a longer recover time. This result implies an asymptotic growth path that increases toward the steady state at a decreasing rate.

With completion of recovery at time $(\tau_1) + T$, the speed of convergence returns from $\boldsymbol{\sigma}$ to σ. The re-adjustment in the speed of convergence renders the growth paths of both K and U(C) still continuous as depicted in Figure 1.

4.2. Path of convergence and recovery time for the case of recurrent disasters

To extend our model, suppose we have recurrent disaster events at times $\tau_1, \tau_2, \tau_3, \ldots \tau_n$, and so forth, that may be non-overlapping or overlapping. After the nth disaster, the extended Ramsey equation (5) is now

$$r_t = F_K - \delta = \eta g + Pe^{-(t-\tau_n)}D_K, t \geq \tau_n \tag{8}$$

where we assume that the hazard rate P and severity of disaster D_K remain constant. Equation (8) is the analogue of equation (5). After natural disaster at time τ_n, the exponential density function in effect starts again at τ_n, hence the $(t - \tau_n)$ in the exponential. The trajectory of capital from time τ_n forward is now approximated by the equation

$$\log[K(t - \tau_n)] = \left[1 - e^{-\sigma(t-\tau_n)}\right] \log[K^*] + \left[e^{-\sigma(t-\tau_n)}\right] \log[(1 - D)K(\tau_n)] \qquad (9)$$

which applies until the next disaster event occurs at time $t = \tau_{(n+1)}$. Equations (8) and (9) require modification by substituting $\tau_{(n+1)}$ for τ_n throughout. These modified equations then apply until yet again the next disaster occurs.

4.2.1 Non-overlapping events

As an extension of Figure 1 might suggest, the optimum trajectory of felicity may be comprised of piecewise continuous segments, with points of discontinuity and downward resets at times of disaster τ_1, τ_2, τ_3, ... τ_n, ... If the number n of recurrent, but non-overlapping disasters is finite, eventually the risk of future disasters disappears, and the scenario appears to resemble a drawn-out single disaster. The scenario of an infinite number of non-overlapping disasters entails an infinite number of resets to the economy on the way to the same, undisturbed steady state. In either case, finite or infinite, each event may be analyzed as an individual disaster, establishing the result that the sustainability criterion is repeatedly satisfied.

4.2.2 Overlapping events

The compound disaster in Fukushima, Japan, serves as a highly prominent example of triple calamities. In 2011, Fukushima Prefecture suffered a compound disaster comprised of three, closely sequential, extremely adverse events: an earthquake, followed by a tsunami, and then a severe nuclear accident. Reporting and commentary since the event have focused on the bad news, often with a misleading slant. But according to recent on-scene accounts and news reports, Fukushima has made substantial recovery, although nuclear radiation levels remain unsafe in a small coastal area of approximately 144 square miles, less than 3% of the Prefecture's total area (5,321 square miles). And by 2040, Fukushima aims to cover 100% of its energy demand with non-nuclear renewable energy, a mode of adaptation, consistent with anti-nuclear sentiment, in a region at high risk of repeat earthquakes and tsunami.[18] Unfortunately, despite almost complete recovery from the compound disaster of 2011, Fukushima Prefecture suffered major damage just again on October 12, 2019, laying in the hazard zone of Typhoon Hagibis.

As the Fukushima scenario illustrates, with high hazard rate P and severity of damage D_K, disasters may be compounded or overlapping in the sense that one or more disasters may strike well before recovery from a previous disaster is complete. The case of recurrent disasters can be especially challenging inasmuch as an infinite horizon with a given hazard rate implies that disasters may occur a countably infinite number of times. Depending on the sequence and severity of such overlapping adverse events, attainment of the golden rule steady state and long run sustainability may or may not be possible.

For investigating overlapping disasters, we first approximate the worst case impact of a finite number of disasters by supposing that they occur near simultaneously. For example, consider a double disaster occurring at say τ_1 and τ_1^+. After the first disaster, capital remaining will be $(1 - D_K)K(\tau_1)$. After the second disaster, capital remaining would be computed as [(1 -

[18] Terada, S., Column: Fukushima thrives after recovering from 2011 earthquake, Honolulu Star Advertiser, 7/22/2019; and Davis, R., A Fukushima Ghost Town Seeks Rebirth Through Renewable Energy, Wall Street Journal, 7/12/2019).

$D_K)K(\tau_1) - D_K(1 - D_K)K(\tau_1)] = [(1 - D_K) - D_K(1 - D_K)]K(\tau_1) = [(1 - D_K)^2]K(\tau_1)$. In the case of a triple disaster, capital remaining would be $[(1 - D_K)^3]K(\tau_1)$. The compound disaster at Fukushima, Japan, is a recent example of this case.

More generally, what about the case of overlapping disasters that do not occur near simultaneously? Suppose after the first disaster, a second disaster of the same magnitude occurs during the first recovery, say when only portion $[(h_1)D_K]K(\tau_1)$ has been restored, where $0 \leq (h_1) \leqslant 1$. At the extremes, $(h_1) = 0$ is the case of a near simultaneous double disaster, as in the case of the 2011 compound disaster in Fukushima; then $(h_1) = 1$ represents the case of adjacent disasters with the second disaster occurring right at time $t = (\tau_1) + T$ of full recovery from the first, as approximated by the very recent strike of Typhoon Hagibis affecting Fukushima Prefecture yet again. Simple algebraic computation yields the expression for capital remaining after occurrence of the second disaster: $\{1 - [1 - (h_1)]D\}(1 - D)K(\tau_1)$. Here again, following the analysis of section 4.1, the sustainability criterion can still be established.

After n overlapping disasters, capital remaining generalizes to $\prod_{i=1}^{n-1}\{1 - [1 - (h_i)]D\}(1 - D)K(\tau_1)$. For any finite n, $\prod_i\{1 - [1 - (h_i)]D\}(1 - D)K(\tau_1) > 0$, and recovery is still possible. Figure 2 plots a simple graph for the case of $n = 5$ with $(h_1) = 1$, $(h_2) = 1/2$, $(h_3) = 1$, and $(h_4) = 0$.

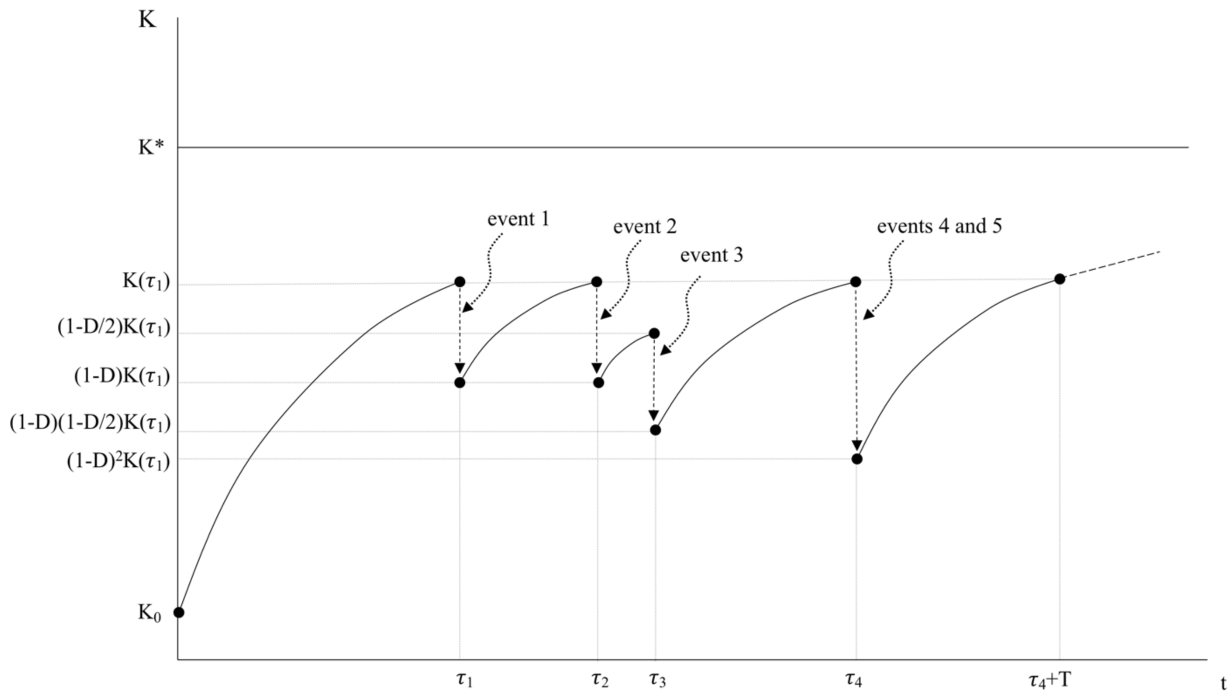

Fig. 2 Five overlapping disasters. $D = \frac{1}{4}$, $h_1 = 1$, $h_2 = \frac{1}{2}$, $h_3 = 1$, $h_4 = 0$; h_i is the portion of disaster recovery attained at time of the next event. As shown, events 1 and 2 represent adjacent disasters; events 4 and 5 comprise a compound, double disaster, which is adjacent to event 3.

The greater challenge to sustainability is the prospect of an infinite number of overlapping natural disasters. As $n \rightarrow \infty$, there are two ways that sustainability may be prevented.

(1) For $(h_i) < 1$ with only finitely many disasters, the trajectory of the capital stock eventually follows a profile, exhibiting endless repetition of disaster, then full recovery, and then immediate disaster again, for all time. The steady state at K* is never attained, even along the optimal path.

(2) With $(h_i) < 1$ for infinitely many disasters, $\prod_i \{1 - [1 - (h_i)]D\}(1 - D)K(\tau_1) \rightarrow 0$. Catastrophe!

Depending on the sequence and severity of such overlapping adverse events, attainment of the golden rule steady state and long run sustainability may or may not be possible.

4.3 Investment in Adaptation

As appropriate to a small economy, we consider adaptation to limit severity of damage if and when the disaster occurs. Anecdotal accounts suggest that adaptation is typically inadequate in vulnerable economies. Forms of adaptation (e.g. in van der Ploeg and de Zeeuw (2019), include sea walls, storm surge barriers, reinforcement of structures, hardening capital (shielding machinery and critical components), improving drainage systems, rerouting roads and rail systems, and relocating existing capital from high risk to lower risk areas. Van der Ploeg and de Zeeuw (2019) present a model of optimal investment in "adaptation capital" in addition to climate-change mitigation via carbon taxation.

For the case of a small economy that regards the hazard rate as exogenous, we formulate a program of adaptation as a tractable extension to the basic disaster model of section 4.1. The program involves a potentially large, initial project of optimal adaptation at time $t = 0$, followed by a simple algorithm for continued adaptation over time, $t > 0$. Section 4.3.1 outlines the description, structure and consequences of a basic investment project, under the assumption that total capital K has value in production only and not intrinsic or amenity value. In Section 4.3.2, the basic model of adaptation is extended to account for disutility of expected damage from natural disaster, leaving solution details of the optimization problem to the appendix. Then in Section 4.3.3, we specify an algorithm or rule for continued adaptation that readily permits modification of the stochastic Ramsey-Koopmans model, developed earlier in Section 4, and then interpret results. We caution here that model results should be interpreted in light of poorly defined economic geography and key parameters, such as the productivity of adaptation, with values that remain largely undetermined.

4.3.1 Initial Project of Optimal Adaptation: Basic Model

We start at time t = 0 with the small economy facing exogenous hazard rate P and capital damage fraction D_o. The initial capital stock is K_o. Adaptation capital Q works to reduce severity of a possible natural disaster by decreasing the capital damage fraction $D(Q) < D_o$, with $D'(Q) < 0$. Unit cost of adaptation capital is θ, so that total cost of adaptation is θQ. Priority is given to investment in adaptation and precautionary investment in productive capital, so additional capital accumulation is postponed until t > 0.

For optimal investment at time t = 0, the planner's problem is formulated as

$$\max_{C} U(C)$$
$$\text{subject to } F(K_o) = C + \delta K_o + K_o P D(Q) + \theta Q, F(K_o) = A(K_o)^{\alpha} \tag{10}$$

We solve the problem in two stages, a process equivalent to setting up a Lagrangian. First, minimize $Z(Q) = [K_o P D(Q) + \theta Q]$, relaxing the feasibility constraint for the economy to maximize the amount of output available for consumption at t = 0. Take $D(Q) = D_o\left[1 - \left(Q^{\beta}\right)/Q_m\right]$, where 0 < β < 1 measures the productivity of adaptation and $Q_m = F(K_o)/\theta$ is the hypothetical maximum amount of adaptation feasible if all output were allocated to the adaptation project. Set $Z'(Q) = [K_o P D_o]\left[-\beta\left(Q^{\beta-1}\right)/Q_m\right] + \theta = 0$. Solving gives

$$Q^* = \{[\theta Q_m]/[\beta K_o P D_o]\}^{1/(\beta-1)} = \{[F(K_o)]/[\beta K_o P D_o]\}^{1/(\beta-1)}$$
$$= \{A/[\beta P D_o]\}^{1/(\beta-1)}(K_o)^{(\alpha-1)/(\beta-1)} \tag{11}$$

Z(Q*) is in fact a minimum since $Z''(Q) = -\beta(\beta-1)[K_o P D_o]\left[\left(Q^{\beta-2}\right)/Q_m\right] > 0$. By taking logs in equation (11), it is easy to show as a theoretical result that optimal adaptation at t = 0 is increasing in β. Empirically however, the productivity of adaptation has not been well ascertained, making actual determination of Q* difficult.

Second, set $C = F(K_o) - \delta K_o - [K_o PD(Q^*) + \theta Q^*]$, thus maximizing U(C). The initial ratio $[Q^*/K_o] = \{A/[\beta PD_o]\}^{1/(\beta-1)}(K_o)^{(\alpha-\beta)/(\beta-1)}$ is the basis of continued adaptation for $t > 0$, maintaining the constant ratio $[Q(t)/K(t)] = [Q^*/K_o]$, thereby preserving the validity of optimally reduced damage fraction $D^* = D(Q^*)$. We investigate this program in Section 4.3.3, where we now set $D_K = D^*$ for $t > 0$.

One added virtue of this basic model is that it yields a closed form solution for Q*, which can be investigated further with comparative static analysis. By taking logs in equation (11), it is fairly easy to show that Q* is increasing in model parameters K_o, P, and D_o without ambiguity. In contrast, Q* is decreasing in unit adaptation cost θ. Calculations are outlined in the appendix. With respect to the parameter β, which measures the productivity of adaptation capital, the comparative static can be ambiguous depending on values of other model parameters. We show in the appendix that if $\{[PD_o(K_o)^{(1-\alpha)}]/A\} \geq 1$, then Q* is increasing in β without ambiguity. This condition is plausibly satisfied in an economy facing significant disaster risk as gauged by (PD_oK_o). Otherwise, the comparative static is ambiguous. In that regard, further research into the theory and empirics of adaptation productivity, as represented by the parameter β in this model, is warranted.

These comparative static results are important to understanding how changes to fundamental model parameters affect optimal damage fraction $D^* = D(Q^*)$, which directly influences severity of natural disasters, recovery times, and optimal levels of precautionary investment. News reports indicate that after Typhoon Hagibis and other recent disasters, Japan is reassessing disaster preparedness and programs of adaptation in light of increased frequency and severity of natural disasters.

4.3.2 Alternative Model of Adaptation: Accounting for Dis-amenity of Expected Damage

In this expanded approach to adaptation, the presumption is that K_o, the initial stock of total capital (the aggregate of produced, human, and natural capital), has some intrinsic value, such as artistic and architectural value of buildings, bridges and other structures, as well as amenity value of natural resources. Expected damage of intrinsic and amenity value of total capital yields disutility, for which we now account.

Optimal adaptation Q** is determined by maximizing total welfare at t = 0, accounting for both felicity of consumption and dis-amenity of expected capital damage from natural disaster, subject to the economy's feasibility constraint. The planner's problem is:

$$\max_{\{C,Q\}} U(C, Q)$$
$$\text{subject to } F(K_o) = C + \{\delta + PD(Q)\}K_o + \theta Q \quad (12)$$

The details of the problem solution are presented in the appendix with the following specifications: $U(C, Q) = \left[-C(t)^{-(\eta-1)}\right] - [K_o PD(Q)]^{\gamma}$, with $\gamma > 2$ to reflect increasing marginal disutility, and $\eta > 1$ as before. Note that U(C, Q) is negative but increasing in both C and Q; in the appendix we verify that U_1 and U_2 are both positive.

In this model, there is now an explicit tradeoff, the marginal rate of substitution, MRS = $\Delta C/\Delta Q < 0$. Accordingly, optimal Q** will now likely exceed the value Q* developed in the simpler model of Section 4.3.1, so that $Q^* < Q^{**} < Q_m$.

As in Section 4.3.1, we use the functional form $D(Q) = D_o\left[1 - \left(Q^{\beta}\right)/Q_m\right]$, where $0 < \beta < 1$ measures the productivity of adaptation capital. Again, Q_m represents the hypothetical maximum level of adaptation feasible in the economy if all output at t = 0 were dedicated to adaptation: $Q_m = F(K_o)/\theta$.

Having determined Q**, the project then involves actual investment at the corresponding level, thereby lowering the capital damage fraction down to D** = D(Q**). We subsequently take D_k = D** for t > 0.

As a technical matter that we don't dwell on, we note that the static optimization process yields initial consumption C_0 and perhaps an adjusted level of initial capital, as some K_0 may have been converted to adaptation capital, so that $K_o+ < K_o$. (For example, rather than being relocated, some structures serving as productive capital in high risk areas may be simply abandoned and left standing as storm surge barriers, though abandonment still entails adaptation cost.) Nonetheless, without loss of generality, we'll refer to the initial capital stock, even if adjusted, as K_o. However, we may encounter a jump in consumption C, likely positive, in moving from static optimization at time t = 0 to dynamic optimization starting at time. We disregard this jump discontinuity as being insignificant compared to a consumption drop due to natural disaster.

4.3.3 A Simple Program of Continuing Adaptation Over Time

We describe a program of continuing adaptation using the optimal Q* derived in the basic model (i.e., without disutility of expected damage). The process and description can directly apply to use of the alternative model (Section 4.3.2) in follow-on program by direct substitution of Q** for Q*.[19]

Having determined Q* at t = 0, we now take D* = D(Q*) as fixed for $t \geq 0$. Use of fixed D* in the subsequent dynamic optimization model requires an accompanying program of

[19] Having accounted for intrinsic and amenity value of capital in the initial adaptation project at t = 0, the planner invokes the follow-on investment algorithm in terms of Q** for t > 0. This process eliminates the need to incorporate U(C, Q) rather than U(C) in the planner's problem, equation (13).

ongoing adaptation as productive capital accumulates. To keep the model tractable, we invoke a simple rule to accommodate this requirement: at each time $t > 0$, invest in adaptation capital Q to maintain the optimum ratio attained at $t = 0$, Q^*/K_o, so that $Q(t) = (Q^*/K_o)K(t)$ and consequently, $\dot{Q} = (Q^*/K_o)\dot{K}$.[20]

Adaptation capital Q may depreciate along with productive capital K at some proportional rate $\delta\varepsilon$, with $0 \leq \varepsilon \leq 1$, and hence, needs replenishment according to $(\delta\varepsilon)(\theta Q^*/K_o)K$. Some adaptation capital, such as sea walls, storm surge barriers, and drainage systems, may not significantly depreciate, in contrast to adaptation associated with reinforced structures and hardened productive capital. For such a case, we take $\varepsilon < 1$.

After completion of the initial adaptation project, based on static optimization with felicity U(C), dynamic optimization begins at time $t = 0^+$ with felicity now in the functional form of Section 4, $U(C(t)) = \left[-C(t)^{-(\eta-1)}\right]$, with $\eta > 1$. Accounting for disutility of expected capital damage in the felicity function is no longer needed since expected capital damage has already been minimized via D* and the ongoing program of adaptation with the simple investment rule.

The planner's problem, the analogue of equation (4) of Section 4, becomes

$$W(t) = \max_C E\left(\int_t^\infty \{U(C(s)) - U(C^*)\}ds\right)$$

$$\text{subject to } \dot{K} = F(K) - (\theta Q^*/K_o)\dot{K} - \{\delta + (\delta\varepsilon)(\theta Q^*/K_o) + Pe^{-Pt}D^*\}K - C \tag{13}$$

[20] In section 4.3.4, we suggest an alternative dynamic equation for Q that might be used in a more complex, though less tractable model. Ideally, as a reward for greater computational complexity, such a model would permit investigation of research questions beyond the scope of this paper, including conditions under which adaptation capital and productive capital can serve as substitutes.

The dynamic equation may be rewritten as

$$\dot{K}[1 + (\theta Q^*/K_o)] = F(K) - \{\delta + (\delta\varepsilon)(\theta Q^*/K_o) + Pe^{-Pt}D^*\}K - C \tag{14}$$

The Hamiltonian for the problem (equation (13)) is then

$$\begin{aligned} H &= [U(C) - U(C^*)] \\ &+\lambda\left\{(1/[1 + (\theta Q^*/K_o)])\{F(K) - \{\delta + (\delta\varepsilon)(\theta Q^*/K_o) + Pe^{-Pt}D^*\}K - C\}\right\} \end{aligned} \tag{15}$$

Problem solution (equation (15)) yields an adjusted shadow price of capital, $\lambda = [1 + (\theta Q^*/K_o)]U'(C)$, accounting for the added cost of required adaptation capital for each unit of physical capital.

Application of the optimal control maximum principle easily leads to a modified Ramsey equation that serves as a first stage result, which includes the risk of disaster but again, as if the actual occurrence of disaster is postponed indefinitely; this equation is the analog of the extended Ramsey equation (6) presented earlier.

$$\begin{aligned} r = (F_K - \delta) &= \eta[1 + (\theta Q^*/K_o)]g + (\delta\varepsilon)(\theta Q^*/K_o) + Pe^{-Pt}D^* \\ &= \eta g + \{\eta(\theta Q^*/K_o)g + (\delta\varepsilon)(\theta Q^*/K_o)\} + \{Pe^{-Pt}D^*\} \end{aligned} \tag{16}$$

With actual occurrence of disaster at time τ_1, the modified Ramsey equation takes on a two-part characterization. For $t < \tau_1$, the Ramsey equation derived above applies. Then for $t > \tau_1$, the stochastic term undergoes a time reset and is now expressed as $Pe^{-P(t-\tau_1)}(D^*)$.

The precautionary effect in equation (16), $\{\eta(\theta Q^*/K_o)g + (\delta\varepsilon)(\theta Q^*/K_o)\} + \{Pe^{-Pt}D^*\}$, is now dual in nature, representing precautionary investment in both productive capital and adaptation capital along the optimal path. The validity of the optimally minimized damage fraction D*, accounting for adaptation cost, is thereby sustained.

We have shown that optimal investment in adaptation yields an optimal damage fraction D*, which is instrumental in limiting severity of natural disasters, decreasing recovery times, and reducing the need for precautionary additions to productive capital. Through comparative static analysis, we have also shown that optimal investment in adaptation increases with parametric increases in K_o, P, D_o, and very plausibly, β, the measure of adaptation productivity, but decreases with unit adaptation cost θ. Accordingly, D* is optimally adjusted in the opposite directions. Another significant finding when specifically accounting for intrinsic and amenity value of total capital (the aggregate of produced, human, and natural capital) is that optimal investment in adaptation, now designated Q**, likely exceeds the level Q* applicable to the basic case without dis-amenity. The condition $Q^{**} \geq Q^{*}$ consequently yields the result that $D^{**} \leq D^{*}$.

4.3.4 An alternative approach for future research

As a closing note, we suggest an alternative formulation of a model for optimal investment in adaptation that offers the potential for addressing issues of interest beyond the scope of this paper. The proposed model retains the formulation of the planner's problem, equation (13), but now makes optimization subject to two dynamic feasibility constraints that are coupled:

$\dot{K} = F(K) - [\delta + Pe^{-Pt}D(Q)]K - \theta Q - C$, with $K(0) = K_o$ and $D'(Q) < 0$. The specification for D(Q) may reasonably differ from that presented in section 4.3.1.

$\dot{Q} = G(K, Q) = \psi[aK^{\rho} + bQ^{\rho}]^{1/\rho} - \delta_q Q$, with $-\infty < \rho \leq 1$ and $Q(0) = Q_o$. One option for Q(0) is to take $Q_o = Q^*$ derived in section 4.3.1. The parameter δ_q is the fixed rate of deprecation for adaptation capital Q, likely differing from the rate of depreciation δ for productive capital.

In the constant elasticity of substitution (CES) specification, ψ may be regarded as an efficiency parameter, with a and b as distribution parameters and ρ, the substitution parameter. (See Varian (1992) or Chung (1994) for good discussions of CES production functions.)

Ideally, this alternative model gives the promise of yielding additional results and insights, including conditions under which adaptation capital and productive capital are viable substitutes, and to what degree. The apparent tradeoff in this approach, however, is an increase in technical complexity and reduction in tractability. Model solution will likely be non-trivial. We leave this challenge as an opportunity for future research.

5. Conclusions

Either inadequate disaster preparation or excessive precaution can undermine the prospect of sustaining inter-temporal welfare, but with optimal preparedness, sustainability is still possible. But even if sustainability is possible, inadequate precautionary investment constrains the economy to follow a suboptimal path with a more painful and burdensome recovery.

Capital-destroying natural disasters do not necessarily alter the steady state of the optimal economy, which depends on the production and utility functions. Perhaps more surprising, the speed of convergence to the steady state, by the conventional measure, modified for stochastic depreciation, is increased by disaster risk. The rate of capital depreciation, fixed plus stochastic, determines the rate at which capital needs to be replenished along the optimal path. The greater the rate of needed replenishment, the greater the amount of investment in the economy, rendering faster adjustment.

A natural disaster restarts the time paths of capital, consumption, and welfare in accordance with the new levels of capital stocks post disaster. In the case of recurrent, non-overlapping disasters, the result is a series of restarts. Despite destruction of capital in the short run, economic recovery can be achieved in the medium run, and the economy can approach the same golden rule steady state if optimal paths are followed. Under these conditions, the optimal paths satisfy the Arrow et al. (2004) sustainability criterion that intertemporal welfare not decline, despite the temporary fall of felicity, U, even in the absence of technological change. A sustainability constraint requiring consumption, planner's utility, or intertemporal welfare to be non-declining would be redundant. In summary, absent highly damaging overlapping events, natural disasters do not make sustainability impossible. The achievement of optimal growth, however, requires precautionary investment in both productive capital and adaptation to disaster risk.

Only in the case of recurrent disasters so frequent and severe as to prevent the recovery of total capital stock would sustainability be rendered impossible. For most economies, especially more developed ones, property losses tend to be a small part of total capital. For example, property damages from the most devastating hurricanes of 2018 (Florence and Michael), while causing property damages of USD 24 and 25 billion respectively (NOAA 2018), were still quite small relative to the size of economies wherein they occurred.

Determining optimal adaptation is difficult, especially given that productivity of adaptation has not been well ascertained. More research is needed in this area. Our model of optimal adaptation yields preliminary results that increasing adaptation reduces severity and recovery times of individual disasters, but should be balanced against adaptation cost to determine the optimum level of investment. For managing recurrent disasters, optimal adaptation, fostering shorter recovery times, reduces the likelihood that multiple events will overlap, threatening the prospect of sustainability. Ideally, our adaptation model becomes a foundation for more complex models that, among other issues, can identify conditions under which adaptive capital and productive capital serve as substitutes. We present the outline of a possible alternative modeling approach in that regard. Nonetheless, in spite of present research gaps, empirical evidence from previous disasters strongly indicates the direction of required policy: greater and earlier investment in adaptation.

Production and utility could be modeled more generally by including labor and leisure as explicit arguments in the production and utility functions. Such models would allow one to investigate how the optimal allocation between production and investment varies with changes in the output elasticity of labor and the weight assigned to labor disutility.

Natural capital can also be put into the model, thus completing the nexus between disaster economics and sustainable development. This would add another pathway for natural disasters to impede sustainability, in particular if the disaster reduces a resource stock below its point of critical depensation. This would also facilitate the extension of green accounting to include the prospect and occurrence of natural disasters.

Another important, if daunting, extension would be to build ecological resource interactions into the model, thus providing more complete economic foundations for ecological resource economics and sustainability science (Clark 2007). Notions of network effects, emergence, spontaneous order, non-linearities, critical transitions, bifurcations, regime shifts, tipping points, and chaos in adaptive, complex systems, though regarded as “faddish” in some circles, are now being incorporated into advanced models. Scheffer (2009) provides an accessible overview of the issues and challenges involved.

Research in this area will likely involve continued development and refinement of models linking sustainability theory, uncertainty, and the economics of natural disaster, including technological change. For example, disaster shocks to productivity could be included in endogenous growth models with implications for business cycle theory. Further research in the realm of stochastic analysis would also help in providing foundations for estimable models with empirically-based disaster severities and probabilities. Most of the sustainable growth literature, as here, is in the context of optimal growth (e.g. Heal's 2003 conclusion that most models of optimal growth yield sustainable growth paths but not the other way around).

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Appendix

Extended Ramsey equation for sustainable growth with risk of natural disaster

We take ρ = 0, and the probability of risk is modeled as an exponential distribution with parameter P and density function Pe^{-Pt}. D_K is the fraction of capital stock damaged by the disaster. The planner's problem is

$$\max_{C} E\left\{\int_{0}^{\infty}[U(C) - U(C^{*})]dt\right\} \tag{A1}$$

subject to

$$\dot{K} = F(K) - \{\delta + [Pe^{-Pt}D_K]\}K - C \text{ and}$$

The current value Hamiltonian corresponding to (A1) is

$$H = [U(C) - U(C^{*})] + \lambda[F(K) - \{\delta + [Pe^{-Pt}D_K]\}K - C] \tag{A2}$$

The standard first order conditions for the optimal control problem (A2) are

$$\partial H/\partial C = U_C - \lambda = 0 \tag{A3}$$

$$\partial H/\partial K = -\partial\lambda/\partial t = \lambda[F_K - \{\delta + [Pe^{-Pt}(D_K)]\}] \tag{A4}$$

From (A3) and (A4) $\dot{\lambda} = \dot{U}_C$ and $\dot{\lambda} = -\lambda[F_K - \{\delta + [Pe^{-Pt}(D_K)]\}]$. Equating expressions for $\dot{\lambda}$ and rearranging yields:

$$-(1/U_C)\dot{U}_C = \eta(1/C)\dot{C} = \eta g = F_K - \{\delta + Pe^{-Pt}(D_K)\}, \tag{A5}$$

where $g = (1/C)\dot{C}$. The extended Ramsey equation with disaster risk is then:

$$r = F_K - \delta = \eta g + Pe^{-Pt}(D_K) \tag{A6}$$

The second term on the RHS of (A6) represents the precautionary effect, inducing investment in precautionary capital.

As $t \rightarrow \infty$ and the economy approaches the steady state, $g \rightarrow 0$, and $Pe^{-Pt} \rightarrow 0$. So in the steady state

$$F_K = \delta \text{ and} \tag{A7}$$

Comparative Statics (Section 4.3.1) for Optimal Adaptation Capital Q*

Start with the result

$$Q^* = \left[\frac{\theta Q_m}{\beta P D_o K_o}\right]^{\frac{1}{(\beta-1)}} = \left[\frac{\beta P D_o K_o}{\theta Q_m}\right]^{\frac{1}{(1-\beta)}} = \left[\frac{\beta P D_o (K_o)^{1-\alpha}}{A}\right]^{\frac{1}{(1-\beta)}},$$

for $Q_m = A(K_o)^\alpha$. It is sufficient to derive comparative statics in terms of logQ*, given that $d \log Q^* = (1/Q^*)dQ^*$, with Q* > 0.

Using the second expression for Q*

$$\log Q^* = \frac{1}{(1-\beta)}\{\log \beta + \log[P D_o K_o] - \log[\theta Q_m]\}$$

Differentiation of this equation readily yields the results that comparative statics (in terms of both logQ* and Q*) are positive for P, D_o and K_o, but negative for θ.

The derivation of the comparative static for β proceeds as follows. Using the third expression for Q*,

$$\log Q^* = \frac{1}{(1-\beta)}\log \beta + \frac{1}{(1-\beta)}\log\left\{\frac{P D_o (K_o)^{1-\alpha}}{A}\right\} = J(\beta) + L(\beta)$$

Differentiating one term at a time leads to:

$$J'(\beta) = \frac{(1-\beta)/\beta + \log \beta}{(1-\beta)^2}$$

In the numerator, make the approximation $\log(1+x) \approx x$ with $x = \beta - 1$. Then

$$\frac{(1-\beta)}{\beta} + \log \beta \approx \frac{(1-\beta)}{\beta} + (\beta - 1) = \frac{(\beta^2 - 2\beta + 1)}{\beta} > 0 \text{ for } \beta < 1$$

Note that the quadratic term is zero for $\beta = 1$ but positive elsewhere. Hence $J'(\beta) > 0$.

Next,

$$L'(\beta) = \frac{1}{(1-\beta)^2}\log\left\{\frac{PD_o(K_o)^{1-\alpha}}{A}\right\}$$

If $\left\{\frac{PD_o(K_o)^{1-\alpha}}{A}\right\} \geq 1$, then $L'(\beta) \geq 0$. Otherwise $L'(\beta) < 0$.

Finally, $d\log Q^*/d\beta = J'(\beta) + L'(\beta)$. If $L'(\beta) \geq 0$, $d\log Q^*/d\beta > 0$; otherwise, the comparative static is ambiguous. For example, take $\alpha = 1/2$, A = 1, P = D_o = 1/4. Then $L'(\beta) \geq 0$ for $K_o \geq 256$; for A = 2, the condition requires $K_o \geq 1024$.

Solution of the Planner's Problem (Section 4.3.2) for Optimal Adaptation at Time t=0

Set $U(C,Q) = \left[-C(t)^{-(\eta-1)}\right] - [K_0PD(Q)]^{\gamma}$ with $\eta > 1$ and $\gamma > 2$. Then

$$U_1 = (\eta-1)C^{-\eta} > 0$$

$$U_2 = -\gamma[K_0PD(Q)]^{\gamma-1}[K_0PD'(Q)] > 0.$$

The planner's problem can be stated as:

$$\max_{\{C,Q\}} U(C,Q)$$

subject to $F(K_0) = C + \{\delta + PD(Q)\}K_0 + \theta Q$

where K_0 is the initial capital stock and θ is the unit cost of adaptation capital. Form the Lagrangian:

$$L = U(C,Q) + \lambda[F(K_0) - C - \{\delta + PD(Q)\}K_0 - \theta Q]$$

The first order conditions are as follows:

$$\partial L/\partial C = U_1 - \lambda = 0$$

$$\partial L/\partial Q = U_2 + \lambda[-K_0 PD'(Q) - \theta] = 0$$

$$\partial L/\partial \lambda = F(K_0) - C - \{\delta + PD(Q)\}K_0 - \theta Q = 0$$

Combining the first two FOCs yields

$$\frac{U_2}{U_1} = K_0 PD'(Q^*) + \theta$$

where $\frac{U_2}{U_1}$ is the negative of the marginal rate of substitution (MRS) between consumption and adaptation ($\Delta C/\Delta Q < 0$) at time $t = 0$. For a solution to exist with $\frac{U_2}{U_1} > 0$, we require that $[K_0 PD'(Q^*) + \theta] > 0$. Using $D(Q) = Do[1 - (Q^{\beta})/Qm]$ with $0 < \beta < 1$ and $Qm = [A(Ko)^{\alpha}]/\theta$, it is easy to show that this condition is satisfied for a reasonable choice of parameters P, Do, α, and β (unit cost θ factors out).

Substituting full expressions for U_1 and U_2 in the term U_2/U_1 and then combining with the first order condition $\partial L/\partial \lambda = 0$ to substitute for C, leads to a challenging system of two nonlinear equations in the two variables C and Q. A numerical approach to the solution of this non-linear system will be required to yield the desired Q**.

One potential approach is the process of iteration, starting with the initial value Q* from the basic model and solving for $C_1 = F(Ko) - \delta Ko - [KoPD(Q^*)]$. Then substituting for C_1 in the system, numerically solve for Q_1. Now set $C_2 = F(Ko) - \delta Ko - [KoPD(Q_1)]$. Continuing the process, generates a chain: $Q^* \to C_1 \to Q_1 \to C_2 \to Q_2 \to C_3 \to Q_3 \to \ldots\ldots$

This process of iteration may lead to one of several outcomes:

1) The process converges to a unique Q** with $Q^* < Q^{**} < Qm$.

2) The process ultimately alternates between values Q^ and Q^^ with Q* < Q^ < Q^^ < Qm, in which case Q** may be approximated as [Q^^ - Q^]/2.
3) The process diverges and fails to yield a numerical approximation for Q**, in which case other methods must be pursued.

For numerical approaches to solution, we refer to Judd (1988), especially chapters 5 (Nonlinear Equations) and 6 (Approximation Methods).